Deciding Hamiltonicity by Max-Flow Min-Cut for cactus-like graphs

Adam Kabela (joint work (in progress) with H. Broersma, H. Qi and E. Vumar)

We recall that a graph is a cactus if every edge is in at most one cycle. For a graph H, we say an H^*-graph is either the graph H or a graph obtained from an H^*-graph by choosing its arbitrary vertex and by adding a false twin of this vertex (that is, the two vertices have the same neighbourhood in the resulting graph; in particular, they are not adjacent).

We show that if H is either a bipartite cactus or an odd cycle and G is an H^*-graph (on at least 3 vertices), then G is Hamiltonian if and only if G is 1-tough.

We use the classical Max-Flow Min-Cut Theorem [5] as the main tool, and we either find a Hamilton cycle in G or a separating set of k vertices whose removal disconnects G into more than k components.

We are interested in what happens if we consider adding true twins (true twins are adjacent) along with adding false twins (the resulting class of graphs is a superclass of P_4-free graphs by [2]). The present result is motivated by a similar result on C_5^*-graphs [4], and the question is motivated by the results of [1], [6] and [3].

References

[1] J. C. Arditti, R. Cori: Hamilton circuits in the comparability graph of a tree, Combinatorial Theory and Its Applications, Proceedings Colloquium, Balatonfüred (1969), 41–53.

[2] H. Bandelt and H. M. Mulder: Distance-hereditary graphs, Journal of Combinatorial Theory, Series B 41 (1986), 182–208.

[3] H. J. Broersma, B. Li and S. Zhang: Forbidden subgraphs for hamiltonicity of 1-tough graphs, Discussiones mathematicae Graph theory 36 (2016), 915-929.

[4] H.J. Broersma, V. Patel, and A. Pyatkin: On toughness and hamiltonicity of $2K_2$-free graphs, Journal of Graph Theory 75 (2014), 244–255.

[5] L. R. Ford, Jr. and D. R. Fulkerson: Maximal flow through a network, Canadian Journal of Mathematics 8 (1956), 399–404.

[6] H. A. Jung: On a class of posets and the corresponding comparability graphs, Journal of Combinatorial Theory, Series B 24 (1978), 125–133.