The $(\Delta + 2)$-conjecture in incidence coloring of graphs

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An *incidence* in a graph G is a pair (v, e) where v is a vertex of G and e is an edge of G incident to v. Two incidences (v, e) and (u, f) are adjacent if at least one of the following holds: (1) $v = u$, (2) $e = f$, or (3) $vu \in \{e, f\}$. An *incidence coloring* of G is a coloring of its incidences assigning distinct colors to adjacent incidences. The originators, Brualdi and Quinn Massey (R. A. Brualdi, J. J. Quinn Massey, Incidence and strong edge colorings of graphs, Discrete Math. 122(1–3) (1993), 51–58), conjectured that every graph G admits an incidence coloring with at most $\Delta(G) + 2$ colors. The conjecture is false in general (B. Guiduli, On incidence coloring and star arboricity of graphs, Discrete Math. 163(1–3) (1997), 275–278), but there are many classes of graphs for which it holds. We will present main results from the field and introduce some of our recent ones. Namely, we will focus on incidence coloring of Cartesian products of graphs (P. Gregor, B. Lužar, and R. Soták, On incidence coloring conjecture in Cartesian products of graphs, Discrete Appl. Math. 213 (2016), 93–100) and subquartic graphs (P. Gregor, B. Lužar, and R. Soták, Note on incidence chromatic number of subquartic graphs, J. Combin. Optim. (2016), published online).