# Conflict-free colorings

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A partial vertex coloring of a hypergraph is *conflict-free* if for every hyperedge $e$ there is a color $c$ such that $e$ contains exactly one vertex colored with $c$. By a *conflict-free coloring* of a graph $G$ we mean a conflict-free coloring of the closed neighborhood hypergraph of $G$ (i.e. it is a partial coloring of vertices of $G$ such that for every vertex $v$ there is a colors that appears exactly once in a closed neighborhood of $v$). The minimum number of colors in such a coloring is called the *conflict-free chromatic number*.

**Problem 1 ([2])** *What is the maximum conflict-free chromatic number of a graph with maximum degree $\Delta$?*

The answer in Problem 1 is at most $O\left(\log^{2+\epsilon}\Delta\right)$ [2, Theorem 1.8]. The problem can be considered in restricted variants: when the graph is $\Delta$-regular and has girth at least $g$ for some constant $g$.

**Problem 2 ([1])** *For which pairs $(r, \Delta)$ the maximum conflict-free chromatic number of an $r$-uniform hypergraph with maximum degree $\Delta$ is at most $\Delta$?*

The answer for problem 2 is positive in the case when $r$ is large enough and $\Delta$ is large enough compared to $r$, and for $r = 4$ [1, Theorem 4]. The question remains open for any $r \geq 5, \Delta \geq 3$.

## References


[1] M. Axenovich and J. Rollin. Brooks Type Results for Conflict-Free Colorings and $\{a, b\}$-factors in graphs. Dicrete Math. 338: 2295–2301, 2015.

[2] J. Pach and G. Tardos. Conflict-free colourings of graphs and hypergraphs. Combinatorics, Probability and Computing, 18(5):819 – 834, 2009.