Some open problems on packing colourings

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Let G be a graph. A packing colouring of G is a colouring of vertices of G such that vertices coloured with colour i must have pairwise distance greater than i. The packing chromatic number, denoted $\chi_\rho(G)$, is the smallest integer k such that G is k-packing colourable. In this talk we present some open problems related to the packing colourings. Since it is quite difficult to state exact values of the packing chromatic number for some special classes of graphs, we mainly look for lower and upper bounds.

One of the classical example is the infinite square grid, i.e., the Cartesian product of two infinite paths. The best known bounds are 13 (lower) and 15 (upper) due to B. Martin, Raimondi, Chen, J. Martin.

Problem 1 *Find the exact value of the packing chromatic number of the infinite square grid.*

Sloper showed that the packing chromatic number of any binary tree (max. degree 3) of arbitrary height is at most 7 while, for the infinite complete ternary tree (vertices of degree 4 and 1 only), the packing chromatic number is unbounded. An interesting question is whether each (sub)cubic graph G has $\chi_\rho(G)$ finite. Balogh, Kostochka and Liu answered this question negatively for the class of cubic graphs. Another question arose (Brešar, Ferme):

Problem 2 *Is the packing chromatic number finite for every subcubic planar graph?*

Since the question seems to be difficult, one can restrict the class of graphs under consideration to subcubic outerplanar or subcubic planar bipartite graphs.

The subdivision graph of a graph G, denoted $S(G)$, is obtained from G by subdividing every edge of G. Gastineau and Togni asked whether the subdivision of any subcubic graph is 5-packing colourable:

Problem 3 *Is it true that* $\chi_\rho(S(G)) \leq 5$ *for every subcubic graph* G*?*

The best known upper bound is 8 due to Balogh, Kostochka and Liu.