On edge colourings avoiding colour sets inclusions

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(joint work with Jakub Kwaśny)

Let $\chi'_{\subset}(G)$ be the least number of colours necessary to properly colour the edges of a graph G with minimum degree $\delta \geq 2$ so that the set of colours incident with every vertex is not contained in a set of colours incident to any its neighbour. We investigate the conjecture that $\chi'_{\subset}(G) \leq \lceil (1+\frac{1}{\delta-1})\Delta \rceil$ for each connected graph G with $\delta \geq 2$ which is not isomorphic to C_5. If proven, this could not be improved. Using a probabilistic argument we support this conjecture by showing that for any fixed $\delta \geq 2$, $\chi'_{\subset}(G) \leq (1+\frac{4}{\delta})\Delta(1+o(1))$ (for $\Delta \to \infty$), what implies that $\chi'_{\subset}(G) \leq (1+\frac{4}{\delta-1})\Delta$ for Δ large enough. The problem remains open though in general and in many intriguing special cases, including e.g. bipartite graphs and subcubic graphs.