Polynomial χ-binding functions for P_5-free graphs

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A graph G with clique number $\omega(G)$ and chromatic number $\chi(G)$ is *perfect* if $\chi(H) = \omega(H)$ for every induced subgraph H of G. A family $\mathcal{G}$ of graphs is called χ-bounded with binding function f if $\chi(G') \leq f(\omega(G'))$ holds whenever $G \in \mathcal{G}$ and G' is an induced subgraph of G. In this talk we will present a survey on polynomial χ-binding functions for (P_5, H)-free graphs.
Moreover, we are interested in the following two problems:

1. Which classes of (P_5, H)-free graphs admit a binding function $f(\omega(G)) = \omega(G) + const.$?

2. It is known that $\chi(G) \leq \omega(G) + 1$ for every $(P_5, diamond)$-free graph and that $\chi(G) \leq \omega(G) + 3$ for every $(P_6, diamond)$-free graph. Is it true that $\chi(G) \leq \omega(G) + const.$ for every $(P_7, diamond)$-free graph?