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University of West Bohemia, Pilsen, Czech Republic,
Union of Czech Mathematicians and Physicists, office Pilsen, Czech Republic

and

School of Information Technology and Mathematical Sciences,
University of Ballarat, Victoria, Australia

International Workshop on Optimal Network Topologies

IWONT 2007

http://iti.zcu.cz/iwont2007

Pilsen-Černice, 17th September - 21st September, 2007

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Proceedings Editor: Roman Kužel

Sponsored by: **Institute for Theoretical Computer Science (ITI), Grant 1M0545 of the Czech Ministry of Education**
Department of Mathematics, University of West Bohemia, Research Plan MSM 4977751301 of the Czech Ministry of Education
School of ITMS, University of Ballarat, Australia

*The workshop logo is based on a drawing of the Hoffman-Singleton graph found by R.A. Litherland.

Programme

Monday, September 17

13:00	Lunch
14:30 - 14:45	Opening
14:45 - 15:25	*Leif K. Jørgensen:* **Upper bounds for the degree/diameter problem**
15:30 - 15:50	*Minh Nguyen:* **On graphs of diameter two and order close to Moore bound**
15:50 - 16:15	Coffee Break
16:15 - 16:45	*Charles Delorme:* **Almost Moore bipartite graphs**
16:50 - 17:10	*Guillermo Pineda-Villavicencio:* **New results on the degree/diameter problem**
17:15 - 17:35	*Jakub Teska:* **Divisibility conditions in almost Moore digraphs**
18:30	Dinner
19:30	Welcome party

Tuesday, September 18

9:00 - 9:30	*Jozef Širáň:* **The large and the small: Covering constructions, this time for near-cages**
9:35 - 9:55	*Xavier Marcote:* **Extending to $(D;g)$-cages some results for $(k;g)$-cages**
9:55 - 10:30	Coffee Break
10:30 - 11:00	*Evelyne Flandrin:* **Cycles in graphs: around the hamiltonian problem**
11:05 - 11:25	*Přemysl Holub:* **Edge-closure concept in claw-free graphs**
11:30 - 12:00	*Sanming Zhou:* **Routing and gossiping in Frobenius graphs**
12:30	Lunch
14:00 - 18:00	Excursion (Kozel castle), Individual discussions
18:30	Dinner
19:30	Individual discussions

Wednesday, September 19

9:00 - 9:30	*Camino Balbuena:* **Incidence matrices of projective planes and of some regular biparite graphs of girth 6 with few vertices**
9:35 - 9:55	*Yuqing Lin:* **The connectivity of Cages**
9:55 - 10:30	Coffee Break
10:30 - 11:00	*Eyal Loz:* **New record graphs in the degree-diameter problem**
11:05 - 11:25	*Martin Knor:* **On radially Moore graphs and digraphs**
11:30 - 11:55	*Nacho López Lorenzo:* **On radially Moore graphs: An overview**
11:55 - 12:05	*Wagner Emanoel Costa:* **Cages and taboo procedures**
12:30	Lunch
14:30 - 18:00	Excursion (Radyně), Individual discussions
18:30	Dinner
19:30	Individual discussions

Thursday, September 20

9:00 - 9:30	*Geoffrey Exoo:* **Two topics related to the degree/diameter problem**
9:35 - 9:55	*Heather Macbeth:* **The degree-diameter problem, and some Cayley graphs on abelian groups**
9:55 - 10:30	Coffee Break
10:30 - 10:50	*Rinovia Simanjuntak:* **Largest planar digraphs**
10:55 - 11:15	*Ljiljana Brankovic:* **Graceful and graceful-like labellings**
11:20 - 11:40	*Andrea Semaničová:* **Regular supermagic graphs**
11:45 - 12:05	*Petr Kovář:* **On supermagic labelings of regular graphs**
12:30	Lunch
14:30 - 18:00	Excursion (Brewery), Individual discussions
18:30	Conference dinner

Friday, September 21

9:00 - 9:30	*Martin Bača:* **Connection between graceful and antimagic labelings**
9:35 - 9:55	*Marián Trenkler:* **On Latin orthogonal hypercubes and magic hypergraphs**
9:55 - 10:25	Coffee Break
10:25 - 10:45	*JianMin Tang:* **An open problem: Superconnectivity of regular digraphs with respect to semigirth and diameter**
10:50 - 11:10	*Roman Čada:* **Open problems in graph factorizations**
11:15 - 11:35	*Tomáš Kaiser:* **Edge connectivity and splitting**
12:00	Lunch

Abstracts

Connection between graceful and antimagic labelings

Martin Bača

A graph labeling is an assignment of integers *(labels)* to the vertices and/or edges of a graph. Within vertex labelings, two main branches can be distinguish: difference labelings that associate each edge of the graph with the difference of the labels of its endpoints, and sum labelings that assign to each edge the addition of the labels of its endpoints. *Graceful* and *edge-antimagic vertex* labelings correspond to these branches, respectively. We study some connections between them. Indeed, we study the conditions that allow us to transform any α-labeling (an special case of graceful labeling) of a tree into an $(a, 1)$- and $(a, 2)$-edge-antimagic vertex labeling.

Incidence matrices of projective planes and of some regular bipartite graphs of girth 6 with few vertices

Camino Balbuena

In this talk, two Latin squares with entries from $\{0, 1, \ldots, n\}$ are defined to be *quasi row-disjoint* if and only if the cartesian product of any two rows contains at most one pair (x, x) with $x \neq 0$. The main result of this work is a method for constructing a family of q mutually quasi row-disjoint Latin squares for q a prime power. From this family we obtain in a very easy way the incidence matrices of projective planes and the incidence matrices of $(q-r)$-regular bipartite graphs of girth 6 and $q^2 - rq - 1$ vertices in each partite set, where q is a prime power and $r = 0, 1, \ldots, q-3$. Moreover, we improve this result for $r = 1$, finding $(q-1)$-regular bipartite graphs of girth 6 with $q^2 - q - 2$ vertices in each partite set. Some of these graphs have the smallest number of vertices known so far among regular graphs with girth 6.

Graceful and graceful-like labellings

Ljiljana Brankovic

A graceful labeling of a finite undirected graph G with n edges is a one-to-one function from the set of vertices of G to the set $\{0, 1, 2, ..., n\}$ such that the induced edge labels are all distinct. This labeling was originally introduced in 1967 by Rosa who also showed that the existence of a graceful labeling of a given graph G with n edges is a sufficient condition for the existence of a cyclic decomposition of a complete graph of order 2n+1 into subgraphs isomorphic to the given graph G. The famous Graceful Tree Conjecture (also known as Ringel-Kotzig, Rosa's, or even Ringel-Kotzig-Rosa conjecture) says that all trees have a graceful labeling. In this talk we explore bipartite graceful and graceful-like labelings of trees.

Cages and taboo procedures

Wagner Emanoel Costa

Constructing Cages is a challenging task, and many graph theory concepts and techniques are involved. After several efforts in the literature, currently there are loose lower and upper bounds. We formulate the problem as an optimisation model, and we applied a taboo search heuristic procedure to approximate cages according to this model. This is an initial experiment to examine potential of operations research techniques (i.e. branch and bound, branch and cut, scatter search procedures) over this construction task. In this talk, we will present the heuristic algorithm and its initial empirical observations. Because the objective function to be optimised is essential for success or failure of this approach, we would like to discuss with audience about possible ways to construct objective functions.

Open problems in graph factorizations

Roman Čada

To decide for a connected graph F on at least 3 vertices and a graph G whether G contains an $\{F\}$-factor is NP-complete. This result was proved by Hell and Kirkpatrick in 1978.

We give an overview of some open problems related to complexity of finding cycle-factors in (di)graphs.

Almost Moore bipartite graphs

Charles Delorme

We consider here bipartite graphs with degree $d \geq 3$ and diameter $D \geq 3$ having 2 vertices less than the bipartite Moore bound.

Some results of existence or non existence are provided.

Two topics related to the degree/diameter problem

Geoffrey Exoo

Topic 1: There has been significant recent progress finding constructions that improve upper bounds on the orders of optimal degree/diameter graphs. Progress on the lower bounds has been slower. We report on several attempts, and a couple of successes, to establish lower bounds that yield new exact values.

Topic 2 (joint work with *R. Jajcay*): A number of best known constructions for the degree/diameter problem, as well as for the related cage problem, are based on finite groups (using Cayley graphs and voltage graphs). For small instances of these problems it is possible to do complete searches over all such graphs. For larger instances, when the search space becomes huge, it would be helpful to be able to identify groups that will be useful in constructing graphs with relatively small diameter (or large girth). We report on a relationship between the derived series of a finite group and its usefulness in the construction of cages and degree/diameter graphs.

Cycles in graphs: around the hamiltonian problem

Evelyne Flandrin

We consider some results related to the hamiltonian problem: particular classes of graphs, cycles of given length, cycles through given vertices or edges. This survey does not pretend to be exhaustive.

Edge-closure concept in claw-free graphs

Přemysl Holub

Ryjáček introduced a closure concept in claw-free graphs based on local completion at a locally connected vertex. Connected graphs A, for which the class of (C, A)-free graphs is stable under the closure, were completely characterized. We introduce a variation of the closure concept based on local completion at a locally connected edge of a claw-free graph. The edge-closure is uniquely determined and preserves the value of the circumference of a graph. We show that the class of (C, A)-free graphs is stable under the edge-closure if $A \in \{H, P_i, N_{i,j,k}, D^m, D^m_{i,j}, M_{i,j,k}\}$. It is proved that the edge-closure of a claw-free graph is the line graph of a multigraph.

Joint work with *Jan Brousek*.

Upper bounds for the degree/diameter problem

Leif K. Jørgensen

The Moore bound is an easy and well known upper bound on the order of a graph with a given degree and diameter. Although it is expected that (with a few exceptions) graphs with order close to the Moore bound do not exist, it is hard to prove such non-existence. We report on the status of non-existence results.

Edge-connectivity and splitting

Tomáš Kaiser

Vertex splitting is a useful tool for establishing properties of graphs of given edge-connectivity. We give a brief overview of the available "splitting lemmas", starting with the classical one due to Fleischner, and show a recent application to a problem on spanning k-walks in 2-edge-connected graphs. The latter is joint work with *R. Kužel*, *H. Li* and *G. Wang*.

On radially Moore digraphs and graphs

Martin Knor

A regular digraph D with radius r and (out- and in-)degree d is **radially Moore** if it has the theoretically maximum number of vertices and its diameter does not exceed $r+1$. We present a simple construction showing that a radially Moore digraph exists for every possible degree d and radius r. Although the problem is easy for digraphs, analogous problem for graphs seems to be difficult. We show some unsuccessful approaches which gave not a general construction for graphs. We welcome any new ideas to attack the problem.

On supermagic labelings of regular graphs

Petr Kovář

A supermagic labeling of a graph $G(V,E)$ is defined as one-to-one mapping from E to the set of integers $1, 2, ..., |E|$ with the property that the weights (sums of the label of the labels of all edges incident to this vertex) are equal to the same constant for all vertices of the graph. Using the terminology introduced in the book by W. Wallis a supermagic labeling is a vertex magic edge labeling.

Plenty of graph classes have been shown to be supermagic, including complete graphs, complete multipartite graphs, Möbious ladders and some circulant graphs (provided they satisfy necessary conditions). Also compositions and Cartesian products of supermagic graphs are supermagic. On the other hand it is still open, whether for every feasible value of r and n there exists an r-regular graph on n vertices.

In this talk we focus on 6-regular circulant graphs. It is one of the constructions used to find examples of r-regular supermagic graphs on n vertices.

Joint work with *Jaroslav Ivančo* and *Andrea Semaničová*.

On the connectivity of cages

Yuqing Lin

Cages were introduced by Tutte in 1947, and have been extensively studied. Most of the work carried out so far has focused on the existence problem, whereas very little is known about the structural properties of (k,g)-cages. In this presentation, I will summarize known results on the connectivity and edge connectivity of cages. Tang *et al.* has conjectured that every $(4,g)$-cage with odd girth is tightly superconnected. Recently we have proved that this conjecture is true. In this talk, we will also present these new results.

New record graphs in the degree-diameter problem

Eyal Loz

With the help of voltage graphs in combination with efficient random computer search many new Degree Diameter record graphs were found. In this talk I will briefly explain how one can use voltage graphs in order to construct big graphs of given degree and diameter.

On radially Moore graphs: an overview

Nacho López Lorenzo

In the context of the degree/diameter problem, Moore graphs (both, undirected and directed) have been extensively studied. It is well known that such extremal graphs only exists in a very few cases. There are several ways to consider graphs “close” to Moore graphs. One of them consists on relaxing the constraint about the vertex eccentricities. In this framework, regular graphs of degree d, radius r, diameter $r+1$ and order equal to the Moore bound $M_{d,r}$ are called radially Moore graphs.
In this talk we will give an overview about the existence and the structure of radially Moore graphs. Although some results are known, many open questions remains unsettled.

Joint work with *Joan Gimbert*.

The degree-diameter problem, and some Cayley graphs on abelian groups

Heather Macbeth

For a given class of graphs and for some fixed diameter and fixed maximal degree, a natural question is to determine the maximal possible order. This so-called *degree-diameter problem*, a basic problem in graph theory, is of interest both mathematically and for its applications to optimisation problems for networks (for example in telecommunications) with nodes of limited physical capacity.

The focus of work I have done has been the degree-diameter problem for *Cayley graphs*. These highly symmetrical graphs arise naturally from the study of finite groups. Group theory hence provides powerful tools for investigation of the properties of Cayley graphs.

For Cayley graphs on abelian groups, computational evidence for small values of d suggests that for most k and $d > k$ the maximal possible order is close to a Moore-like upper bound, approximately $\frac{d^k}{k!}$. However, the best known constructions valid for infinitely many values of d yield Cayley graphs of significantly smaller order. For the special case of Cayley graphs on abelian groups and of diameter two, computational evidence still suggests that the maximal possible order is close to this theoretical bound $1 + d + \frac{1}{2}d^2$. However, the previous best construction valid for infinitely many values of d produced graphs of order approximately $\frac{d^2}{3}$.

Here, inspired by the observation that both the additive and multiplicative groups of a Galois field are abelian, we outline a new construction for Cayley graphs on abelian groups with diameter two and degree from an infinite set of positive integers d. Graphs in this family have order close to $\frac{3d^2}{8}$. A similar construction, also new, yields Cayley graphs on cyclic groups with diameter two and order close to $\frac{d^2}{3}$.

Joint work with *Jozef Širáň*.

Extending to $(D;g)$-cages some results for $(k;g)$-cages

Xavier Marcote

A $(k;g)$-*cage* is a k-regular graph of girth g and the minimum possible number of vertices. These graphs were introduced by Tutte [A family of cubical graphs, Proc. Cambridge Philos. Soc. (1947) 459–474], and their existence for all values of $k \geq 2$ and $g \geq 3$ was proved by Erdős and Sachs [Regulare Graphen gegebener Taillenweite mit minimaler Knotenzahl, Wiss. Z. Uni. Halle (Math. Nat.) 12 (1963) 251–257]. Even though large amount of work has been carried out on the order of $(k;g)$-cages (especially on sharpening the best lower and upper bounds for the order), very little is known so far about structural properties of these graphs. Despite of it, in this work we take one step further by approaching this kind of properties in a family of graphs that generalize $(k;g)$-cages: these graphs are called $(D;g)$-*cages* (a $(D;g)$-cage is a graph of degree set D, girth g and the minimum possible order), and their existence was proved by Chartrand, Gould and Kapoor [Graphs with prescribed degree sets and girth, Periodica Mathematica Hungarica, 12 (1981) 261–266]. Our goal is to extend to $(D;g)$-cages some basic results that are known for $(k;g)$-cages: for some specific choices of D and g we approach questions as the monotonicity of the order of $(D;g)$-cages, the upper-bounding of their diameter, and their connectivity.

Joint work with *Camino Balbuena*.

On graphs of diameter two and order close to Moore bound

Minh Nguyen

In this talk we shall discuss a structural property of maximal repeats in regular graphs of diameter two which can be used to study such graphs whose orders are close to Moore bound.

New results on the degree/diameter problem

Guillermo Pineda-Villavicencio

We consider the degree/diameter problem both for general graphs and for bipartite graphs.

Graphs of maximum degree 3, diameter $D \geq 2$ and 4 vertices less than the Moore bound are studied and some necessary conditions for their existence are derived.

Furthermore, bipartite graphs with two vertices less than the bipartite Moore bound are considered. For diameter 3, we prove that if such a graph exists, the degree Δ or $\Delta - 2$ is a perfect square. For any diameter, we find formulas to compute the eigenvalues of such graphs and their respective multiplicities.

Joint work with *Mirka Miller*.

Regular supermagic graphs

Andrea Semaničová

A graph is called supermagic if it admits a labelling of the edges by pairwise different consecutive integers such that the sum of the labels of the edges incident with a vertex is independent on the particular vertex. We proved some necessary conditions for existence of supermagic graph of order n and size ε. In general it is difficult to decide for which combination of numbers n and ε there exist a supermagic graph with desired parameters. In this talk we will solve this problem for regular graphs, i.e. we will deal with the existence of an r–regular supermagic graph of order n. We prove that for odd–regular supermagic graphs the necessary condition is also sufficient. We describe some constructions of even–regular supermagic graphs.

Joint work with *Jaroslav Ivančo* and *Petr Kovář*.

Largest planar digraphs

Rinovia Simanjuntak

In this talk, we consider the *degree/diameter* problem for planar digraphs. We give upper and lower bounds for the largest possible order of planar digraphs.

Joint work with *Mirka Miller*.

The large and the small: Covering constructions, this time for near-cages

Jozef Širáň

We show that modifications of known covering constructions for 'large' vertex-transitive and Cayley graphs of diameter two give 'small' vertex-transitive and Cayley graphs of a given degree and girth 5 or 6.

An open problem: Superconnectivity of regular digraphs with respect to semigirth and diameter

Jianmin Tang

The superconnectivity κ_1 of a connected digraph G is defined as the minimum cardinality of a vertex-cut over all vertex-cuts F such that no vertex $f \notin F$ has all its neighbors in F. In this paper we propose an open problem: for any δ-regular digraph of diameter D that $\kappa_1 > \delta$ if $D \leq 2\ell - 1$ and $\ell \geq 2$ where semigirth ℓ is a parameter related with the number of short paths in G. If this open problem is ture, as a corollary, superconnected iterated line digraphs are characterized.

Joint work with *Camino Balbuena, Yuqing Lin* and *Mirka Miller*.

Divisibility conditions in almost Moore digraphs

Jakub Teska

Moore digraph is a digraph with maximum out-degree d, diameter k and order $M_{d,k} = 1+d+\ldots+d^k$. Moore digraphs exist only in trivial cases if $d = 1$ (i.e., directed cycle C_k) or $k = 1$ (i.e., complete symmetric digraph). Almost Moore digraphs are digraphs of order one less than Moore bound. We shall present new properties of almost Moore digraphs from which we prove the nonexistence of almost Moore digraphs for infinitely many values of k and d.

Joint work with *Roman Kužel* and *Mirka Miller*.

On orthogonal latin p-dimensional cubes and magic hypercubes

Marián Trenkler

Orthogonal Latin squares are of importance in many areas of science and its applications. In 1960, R.C.Bose, S.S.Shikhande and E.T.Parker [1] proved that two orthogonal Latin squares of order n exist if and only if $n \neq 2, 6$.

A generalization of Latin squares is Latin p-dimensional cubes (sometimes called *Latin hypercubes*).

We give a construction of p orthogonal Latin p-dimensional cubes (or Latin hypercubes) of order n for every natural numbers $n \neq 2, 6$ and $p \geq 2$. We show the result generalizes this well know result about orthogonal Latin squares.

Definition. *A Latin p-dimensional cube of order n is a p-dimensional matrix*

$$\boldsymbol{Q}^{p,n} = |\boldsymbol{q}(i_1, i_2, \ldots, i_p);\ 1 \leq i_1, i_2, \ldots, i_p \leq n|,$$

such that every row is a permutation of the set of natural numbers $1, 2, \ldots, n$. By a row of $\boldsymbol{Q}^{p,n}$ we mean an n-tuple of elements $\boldsymbol{q}(i_1, i_2, \ldots, i_p)$ which have identical coordinates at $p-1$ places.

Definition. *A p-tuple of Latin p-dimensional cubes*

$$[\boldsymbol{Q}_k^{p,n} = |\boldsymbol{q}_k(i_1, i_2, \ldots, i_p);\ 1 \leq i_1, i_2, \ldots, i_p \leq n|, \quad k = 1, 2, \ldots, p]$$

of order n is called orthogonal, if whenever $i_1, i_2, \ldots, i_p, i'_1, \ldots, i'_p \in \{1, 2, \ldots, n\}$ are such that $\mathbf{q}_k(i_1, i_2, \ldots, i_p) = \boldsymbol{q}_k(i'_1, i'_2, \ldots, i'_p)$ forall *$k = 1, 2, \ldots, p$, then we must have $i_k = i'_k$ for all $k = 1, 2, \ldots, p$.*

The construction of a p-tuple of orthogonal Latin p-dimensional cubes is contained in the proof of the following theorem.

Theorem. *A p-tuple of orthogonal Latin p-dimensional cubes $\boldsymbol{Q}_k^{p,n}$ of order n exists for every natural number $n \neq 2, 6$ and every natural number $p \geq 2$.*

Constructions of Latin hypercubes. Let $\mathbf{R}^n = |\mathbf{r}(i_1, i_2);\ 1 \leq i_1, i_2 \leq n|$ and $\mathbf{S}^n = |\mathbf{s}(i_1, i_2)|$ are two orthogonal Latin squares of order n. The k-th cube arises using the square $\mathbf{R}^n$ $(k-1)$-times and the square $\mathbf{S}^n$ $(p-k)$-times.

We define the k-th Latin p-dimensional cube (for every $1 \leq k \leq p$)

$$\mathbf{Q}_k^{p,n} = |\mathbf{q}_k(i_1, i_2, \dots, i_p)|$$

of order n by the following relation

$$\mathbf{q}_k(i_1, \dots, i_p) = \mathbf{r}(i_1, \mathbf{r}(i_2, \mathbf{r}(i_3, \dots, \mathbf{r}(i_{k-1}, \mathbf{s}(i_k, \mathbf{s}(i_{k+1}, \dots, \mathbf{s}(i_{p-2}, \mathbf{s}(i_{p-1}, i_p)) \dots))) \dots)))$$

for every $1 \leq i_1, i_2, \dots, i_p \leq n$.

Problem. The construction is based on a couple of orthogonal Latin squares and so we give no information about Latin p-dimensional cubes of order 2 and 6. Does there exist p-dimensional Latin cubes for the parameters $n = 6$ and $p \geq 3$?

A generalization of magic squares is magic hypercubes. Since 18th century we know de la Hire's construction of magic square of odd order n from a couple of orthogonal Latin squares. Similary we can made a magic hypercube of odd order n.

If n is odd, then $\mathbf{R}^n = |\mathbf{r}(i_1, i_2) = (i_1 + i_2) \pmod{n};\ 1 \leq i_1, i_2 \leq n|$ and $\mathbf{S}^n = |\mathbf{s}(i_1, i_2) = (i_1 - i_2) \pmod{n}|$ are mutually orthogonal Latin squares. Using these two squares the formula for making a magic p-dimensional cube of odd order was derived. (See [2].)

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Routing and gossiping in Frobenius graphs

Sanming Zhou

What graphs should we use to model interconnection networks in order to achieve high efficiency? Of course the answer to this question depends on how we measure efficiency of a network. In this talk we will focus on efficiency of a network with respect to routing and gossiping. We will show that certain Cayley graphs over Frobenius groups admit "perfect" routing and gossiping schemes in some sense. In particular, the undirected triple-loop network with given diameter and maximum order admit such schemes.

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HYMNA TEORIE GRAFŮ

THE GRAPH THEORY HYMN

HYMNUS AUF DIE GRAPHENTHEORIE

HYMN TEORII GRAFÓW

A GRÁFELMÉLET HIMNUSZA

DIE GRAFIEKTEORIELIED

HIMNO DE LA GRAFOTEORIO

ГІМН ТЕОРІЇ ГРАФІВ

HYMNE TEORI GRAF

HYMNE DE LA THEORIE DES GRAPHES

Text by Bohdan Zelinka
Music by Zdeněk Ryjáček

English text by Donald A. Preece
Deutscher Text von Anja Pruchnewski
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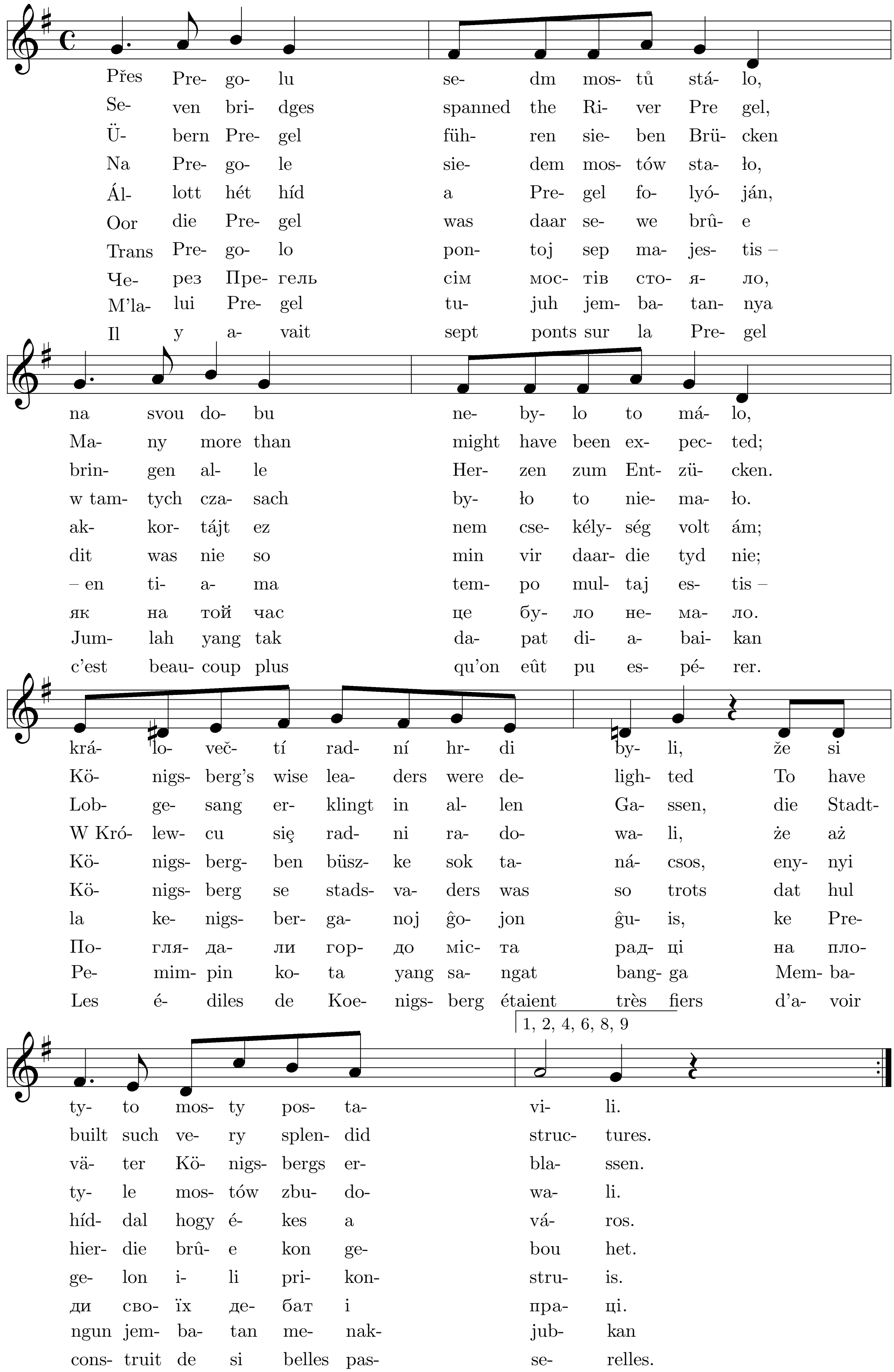
Přes Pre- go- lu se- dm mos- tů stá- lo,
Se- ven bri- dges spanned the Ri- ver Pre gel,
Ü- bern Pre- gel füh- ren sie- ben Brü- cken
Na Pre- go- le sie- dem mos- tów sta- ło,
Ál- lott hét híd a Pre- gel fo- lyó- ján,
Oor die Pre- gel was daar se- we brû- e
Trans Pre- go- lo pon- toj sep ma- jes- tis –
Че- рез Пре- гель сім мос- тів сто- я- ло,
M’la- lui Pre- gel tu- juh jem- ba- tan- nya
Il y a- vait sept ponts sur la Pre- gel
na svou do- bu ne- by- lo to má- lo,
Ma- ny more than might have been ex- pec- ted;
brin- gen al- le Her- zen zum Ent- zü- cken.
w tam- tych cza- sach by- ło to nie- ma- ło.
ak- kor- tájt ez nem cse- kély- ség volt ám;
dit was nie so min vir daar- die tyd nie;
– en ti- a- ma tem- po mul- taj es- tis –
як на той час це бу- ло не- ма- ло.
Jum- lah yang tak da- pat di- a- bai- kan
c’est beau- coup plus qu’on eût pu es- pé- rer.
krá- lo- več- tí rad- ní hr- di by- li, že si
Kö- nigs- berg’s wise lea- ders were de- ligh- ted To have
Lob- ge- sang er- klingt in al- len Ga- ssen, die Stadt-
W Kró- lew- cu się rad- ni ra- do- wa- li, że aż
Kö- nigs- berg- ben büsz- ke sok ta- ná- csos, eny- nyi
Kö- nigs- berg se stads- va- ders was so trots dat hul
la ke- nigs- ber- ga- noj ĝo- jon ĝu- is, ke Pre-
По- гля- да- ли гор- до міс- та рад- ці на пло-
Pe- mim- pin ko- ta yang sa- ngat bang- ga Mem- ba-
Les é- diles de Koe- nigs- berg étaient très fiers d’a- voir
1, 2, 4, 6, 8, 9
ty- to mos- ty pos- ta- vi- li.
built such ve- ry splen- did struc- tures.
vä- ter Kö- nigs- bergs er- bla- ssen.
ty- le mos- tów zbu- do- wa- li.
híd- dal hogy é- kes a vá- ros.
hier- die brû- e kon ge- bou het.
ge- lon i- li pri- kon- stru- is.
ди сво- їх де- бат і пра- ці.
ngun jem- ba- tan me- nak- jub- kan
cons- truit de si belles pas- se- relles.


3, 5, 7
Refrén
Refrain
Eu- le- rův graf všech- ny stup- ně
Eu- le- rian graphs all have this re-
Eu- ler- scher Graph, Dir ist stets zu
Eu- le- ra graf, to fakt o- czy-
Eu- le- ri gráf: min- den fo- ka
Die stel- ling sê: Eu- ler- se gra-
En Eu- ler- -a gra- fo es- tas
Ей- ле- ра граф ма- є сут- ність
Eu- le- rian graph, de- ra- jat- nya
Les graphes d'Eu- ler sont tous de de-
su- dé má — ta vě- ta vždyc- ky pla- tit
stric- tion: The de- gree of a- ny point is
ei- gen, daß sich die Kno- ten grad- gera- dig
wis- ty, wszy- stkie węz- ly są stop- ni pa-
pá- ros, és a té- tel mind- ö- rök- re
fie- ke het by al die pun- te e- we
pa- ra ĉi- u grad' – jen fak- to se- ne-
гар- ну, що всі точ- ки ма- ють сту- пінь
ge- nap Fak- ta i- ni kan se- la- lu
gré pair, c'est un ré- sul- tat tou- jours va-
bu- de; nej- star- ší to ze všech
e- ven. That's the ol- dest graph re-
zei- gen. Es hat die- ser ers- te
rzys- tych. Dos- ko- na- le zna- na
áll most; grá- fok- ról ez ál- lí-
gra- de. Dis die oud- ste re- sul-
ra- ra. Jen la plej mal- no- va
пар- ну. Це най- пер- ший ре- зуль-
be- nar Te- o- re- ma ter- tu-
li- de, c'est le plus vieux thé- o-
vět o gra- fech, jež poz- nal svět.
sult That man- kind has e- ver known.
Satz im Buch der Gra- phen seinen Platz.
jest o gra- fach to pier- wsza z tez.
tás a vi- lág- nak ős- for- rás.
taat oor gra- fie- ke wat ons ken.
tez', sed va- li- das ĝi sen ĉes'.
тат, в кни- гу гра- фів цін- ний вклад.
a Di du- ni- a graf ki- ta
rème qu'on con- nait dans ce do- maine.

1. Přes Pregolu sedm mostů stálo,
na svou dobu nebylo to málo,
královečtí radní hrdi byli,
že si tyto mosty postavili.

2. V podvečeru k řece davy spějí,
po mostech se sem tam procházejí,
otázka jim jedna vrtá hlavou,
jak by měli zvolit cestu pravou.

3. Přes most každý jednou chtějí jíti,
pak se domů zase navrátiti;
nějak jim to ale nevychází,
jeden most vždy přebývá či schází.

Ref. Eulerův graf všechny stupně sudé
má–ta věta vždycky platit bude;
nejstarší to ze všech vět
o grafech, jež poznal svět.

4. Vzpomněli si, muž že v městě žije,
nad jiné jenž velmi učený je,
měřictví i počtů mistr pravý;
musí vzejít rada z jeho hlavy.

5. Mistr Euler smutně hlavou kroutí:
“Jednou cestou nelze obsáhnouti
mostů všech, jak panstvo sobě žádá.
Nepomůže tady žádná rada.

Ref. Eulerův graf . . .

6. Zákony má přece svoje věda,
proti nim se počíti nic nedá.
Mosty ani vodní živel dravý
do cesty se vědě nepostaví.”

7. Když se vojna přes Pregolu hnala,
její bouře mosty rozmetala.
Eulerovo jméno u té řeky
přežilo však mnohé lidské věky.

Ref. Eulerův graf . . .

8. Eulerovo jméno stále žije
dokud žije grafů teorie.
A čim více ubíhají léta,
tím víc tato teorie vzkvétá.

9. Kolegové, naplňme své číše,
k přípitku je zvedněm všichni výše,
at se nám tu stále více vzmáhá
teorie grafů naše drahá.

1. Seven bridges spanned the River Pregel,
Many more than might have been expected;
Königsberg’s wise leaders were delighted
To have built such very splendid structures.

2. Crowds each ev’ning surged towards the river,
People walked bemused across the bridges,
Pondering a simple-sounding challenge
Which defeated them and left them puzzled.

3. Here’s the problem; see if you can solve it!
Try it out at home an scraps of paper!
Starting out and ending at the same spot,
You must cross each bridge just once each ev’ning.

Ref. Eulerian graphs all have this restriction:
The degree of any point is even.
That’s the oldest graph result
That mankind has ever known.

4. All the folk in Königsberg were frantic!
All their efforts ended up in failure!
Happily, a learn-ed math’matician
Had his house right there within the city.

5. Euler’s mind was equal to the problem:
“Ah”, he said, “You’re bound to be disheartened.
Crossing each bridge only once per outing
Can’t be done, I truly do assure you.”

Ref. Eulerian graphs . . .

6. Laws of Nature never can be altered,
We can’d change them, even if we wish to.
Nor can flooded rivers or great bridges
Interfere with scientific progress.

7. War brought strife and ruin to the Pregel;
Bombs destroyed those seven splendid bridges.
Euler’s name and fame will, notwithstanding,
Be recalled with Königsberg’s for ever.

Ref. Eulerian graphs . . .

8. Thanks to Euler, Graph The͡ory is thriving.
Year by year it flourishes and blossoms,
Fertilising much of mathematics
And so rich in all its applications.

9. Colleagues, let us fill up all our glasses!
Colleagues, let us raise them now to toast the
Greatness and the everlasting glory
Of our Graph The͡ory, which we love dearly!

1. Übern Pregel führen sieben Brücken,
   bringen alle Herzen zum Entzücken.
   Lobgesang erklingt in allen Gassen,
   die Stadtväter Königsbergs erblassen.

2. Jeden Abend ström' die Leut' zum Flusse,
   enthusiastisch wimmeln sie voll Muße
   hin und her und quer und 'rum im Kreise,
   um zu lösen ein Problem ganz weise.

3. Und nun hört die Frage aller Fragen.
   Sag, was würdest Du uns dazu sagen!
   Gibt's 'nen Weg, der über jede Brücke
   einmal führt genau und dann zurücke?

Ref. Eulerscher Graph, Dir ist stets zu eigen,
   daß sich die Knoten grad-geradig zeigen.
   Es hat dieser erste Satz
   im Buch der Graphen seinen Platz.

4. Wer begann nach einem Weg zu suchen,
   fand kein Ende, fing bald an zu fluchen.
   Einer, der in diesem Städtchen wohnte,
   brachte die Idee, die sich dann lohnte.

5. Meister Euler fiel sogleich der Groschen:
   "Volk, zerrennt Euch doch nicht die Galoschen!
   Solch ein Brückengang ist niemals machbar,
   der Beweis hier zeige es Euch ganz klar."

Ref. Eulerscher Graph, ...

6. Der Natur Gesetze sind gegeben,
   es umgeht sie keiner Macht Bestreben.
   Weder Brücken noch des Wassers Fließen
   könn' den Weg der Wissenschaft verdrießen.

7. Mit dem Kriege folgt dem Fluß Verderben,
   alle Pracht der Brücken schlug in Scherben.
   Eulers Ruf und Name wird auf Zeiten
   die Geschichte Königsbergs begleiten.

Ref. Eulerscher Graph, ...

8. Dank Dir, Euler, blühend hat mit Wonnen
   Graphenwissenschaft den Start genommen.
   Vielfältigst nutzt man sie mit Fanatik,
   sie bereichert unsre Mathematik.

9. Freunde, laßt uns heut' vom Weine leben,
   Gläser füllen, klingen und erheben.
   Graphentheorie, oh, schätzt sie alle,
   dreimal hoch leb' sie, in jedem Falle!

1. Na Pregole siedem mostów stało,
   w tamtych czasach było to niemało.
   W Królewcu się radni radowali,
   że aż tyle mostów zbudowali.

2. Jak co wieczór tłumy wyruszyły,
   bo nad rzeką spacer bardzo miły.
   Wciąż myśl jedna im zaprząta głowę,
   jak tu wybra c tę właściwą drogę.

3. Przez most każdy raz przejść nie wracając,
   znów się w domu znaleźć nie zbaczając.
   Jakoś im to wcale nie wychodzi,
   most zostaje lub brakuje w drodze.

Ref. Eulera graf, to fakt oczywisty,
   wszystkie węzły są stopni parzystych.
   Doskonale znana jest
   o grafach to pierwsza z tez.

4. Aż nareszcie przypomnieli sobie
   o człowieku żyjącym w ich grodzie,
   Mistrzu geometrii i rachunków,
   On podpowie w którym iść kierunku.

5. Ale Euler smutnie kręci głową,
   bo odpowiedź na to ma gotową:
   "Jedna ścieżka nie wystarczy, aby
   pokryć mosty - nie ma na to rady."

Ref. Eulera graf ...

6. Nie pomogą tutaj dobre chęci,
   nic w nauce nie da się pokręcić.
   Mostów nowych nikt nie wybuduje,
   wodny żywioł tym co są – daruje.

7. Kiedy wojna przez Pregołę gnała,
   mosty wszystkie z ziemią wyrównała.
   Jednak imię Mistrza nad tą rzeką
   przeżyło już wiele długich wieków.

Ref. Eulera graf ...

8. Nowej wiedzy Euler dał podstawy,
   przez co zyskał całe wieki sławy.
   My śladami Mistrza podążamy
   i naukę Jego rozwijamy.

9. Więc, Koledzy, na koniec powsta/nmy.
   Wznosząc toast głośno tak śpiewajmy:
   Niechaj żyje nam Teoria Grafów,
   obwieszczajmy ją całemu światu.

1. Állott hét híd a Pregel folyóján,
   akkortájt ez nem csekélység volt ám;
   Königsbergben büszke sok tanácsos,
   ennyi híddal hogy ékes a város.

2. Alkonyatkor kavarog a népség,
   és fejükben hánytorog a kétség:
   hogy' lehetne jó utat találni,
   minden hídon egyszer általjárni.

3. Mind a hét híd egyszer essen útba,
   séta végén otthon lenni újra;
   de a jó út valahol hibázik,
   egy híd mindig fölös vagy hiányzik.

Ref. Euleri gráf: minden foka páros,
   és a tétel mindörökre áll most;
   gráfokról ez állítás
   a világnak ősforrás.

4. Él egy ember, gondoljunk csak rája,
   itt minálunk, nincs tudásban párja;
   úgy érti a számolást és mérést,
   hogy elébe kell tárni a kérdést.

5. Euler mester fejét búsan rázza:
   "Oly talány ez, nincsen megoldása;
   nincs oly út, mint uraságtok kérik,
   amely minden hidat egyszer érint.

Ref. Euleri gráf: ...

6. Érckemény a tudományos tétel,
   mit sem kezdhet ellene a kétely;
   árad a víz, szilárd a híd rajta,
   még erősb a tudomány hatalma."

7. Háború jött a Pregel folyóra,
   minden hídját ízzé-porrá szórta;
   nemzedékek hosszú során fénylik
   Euler és a folyó neve végig.

Ref. Euleri gráf: ...

8. Euler híre nem ér addig véget,
   míg csak élni fog a gráfelmélet;
   s egyik évre amint jön a másik,
   az elmélet mind jobban virágzik.

9. Jó kollégák, töltsük meg a kelyhet,
   áldomásra mind emeljük feljebb:
   nekünk a gráfelmélet oly drága,
   hadd teremjen sok-sok szép virága!

1. Oor die Pregel was daar sewe brûe
   dit was nie so min vir daardie tyd nie;
   Königsberg se stadsvaders was so trots
   dat hul hierdie brûe kon gebou het.

2. Teen die aand dan wandel al die mense
   oor die brûe het hul loop en wonder,
   oor 'n vraag wat steeds by hul bly spook het
   oor die roete waar hul langs geloop het.

3. Elke brug moet net een maal gebruik word
   en die roete moet dan weer tuis eindig;
   maar dit wou maar net nooit reg uitwerk nie
   want die brûe was nie reg geplaas nie.

Ref. Die stelling sê: Eulerse grafieke
   het by al die punte ewe grade.
   Dis die oudste resultaat
   oor grafieke wat ons ken.

4. Hul onthou toe van 'n man wat daar woon
   met geleerdheid, meer as ander mense –
   Meester van die meetkunde en nog meer –
   hy moes oor die groot probleem nou raad gee.

5. Meester Euler moes hul toe dit meedeel:
   "Dis onmoontlik in 'n enkel roete
   al die brûe een maal oor te wandel;
   daar's geen raad wat hiervoor sal kan help nie."

Ref. Die stelling sê: ...

6. Die natuur het mos sy eie wette
   dis nie moontlik om hul teen te gaan nie,
   nóg die brûe nóg die wilde waters
   kan die wetenskap se gang versteur nie.

7. Toe die oorlog oorspoel daar na Pregel
   is die brûe in die slag vernietig
   maar die naam van Euler sal bly voortleef
   vir nog baie jare by die Pregel.

Ref. Die stelling sê: ...

8. Met die stelling word sy naam verewig
   soos Grafiekteorie sal dit bly lewe
   jaar na jaar kom nuwe resultate
   wat die groei en bloei daarvan bevestig.

9. Vriende kom ons vul nou al ons glase
   vriende kom ons drink nou hierdie heildronk
   en ons hoop vir groei en sterkte voortaan
   vir Grafiekteorie wat ons so lief het!

1. Trans Pregolo pontoj sep majestis –
– en tiama tempo multaj estis –
la kenigsberganoj ĝojon ĝuis,
ke Pregelon ili prikonstruis.

2. Ĉiutage antaŭ la vespero
la urbanoj venas al rivero.
Ĉiam ilin ĝenas la problemo,
kia estu la promen-sistemo.

3. Ili volas pontojn sep transiri,
poste hejmen siajn paŝojn stiri
ofte ili fari tion provas,
sed neniam ili solvon trovas.

Ref. En Euler-a grafo estas para
ĉiu grad' – jen fakto senerara.
Jen la plej malnova tez',
sed validas ĝi sen ĉes'.

4. Ili scias, ke en certa domo
vivas iu scioplena homo;
vera majstro de matematiko:
helpos de la sciencist' logiko.

5. Majstro Euler sian kapon skuas:
"La matematiko nin instruas:
tia voj' sep pontojn ne entenas.
Jen – rezulton do ni ne divenas.

Ref. En Euler-a . . .

6. Siajn leĝojn havas la scienco,
nei ilin estas ja sen senco.
Pontoj eĉ inundoj de l'rivero
ne haltigos marŝon de la vero."

7. La milito Kenigsbergon skuis,
ĝiaj ŝtormoj pontojn sep detruis.
Sed la nomo Euler ĉe l' rivero
vivas dum longega homa ero.

Ref. En Euler-a . . .

8. Ĉiam vivu tiu ci genio,
dum ekzistos grafoteorio.
Kvankam tre rapide tempo fluas,
tiu teorio evoluas.

9. Gekolegoj, glasojn ni plenigu,
por la tosto ĉiujn ni instigu;
vivu en estonta historio
nia kara grafoteorio.

1. Через Прегель сім мостів стояло,
як на той час це було немало.
Поглядали гордо міста радці
на плоди своїх дебат і праці.

2. Кожен вечір юрби йшли до річки,
по мостах бродити завжди слічно,
та не мали спокою од того,
бо шукали правильну дорогу:

3. всі мости ті по разу одному
перейти й вернутися додому.
Та задачка ця не піддається:
то той двічі, то якийсь минеться.

Пр. Ейлера граф має сутність гарну,
що всі точки мають ступінь парну.
Це – найперший результат,
в книгу графів цінний вклад.

4. Та згадали, що між ними чемно
проживає славний муж учений,
він рахує й міряє все радо,
дасть він точно цій проблемі раду.

5. Але Ейлер крутить головою:
"Не пройти все ходкою одною,
як ви це собі запланували,
хоч би ви роками там блукали."

Пр. Ейлера граф . . .

6. Теорему цю вже не змінити,
це – законів непохитний злиток.
Ні потопи, ні великі мóсти
не зупинять науковий поступ.

7. Як війна зла через Прегель гнала,
то мости з землею порівняла.
Им'я ж Ейлера понад рікою
не поховане віків юрбою.

Пр. Ейлера граф . . .

8. Нову галузь, Ейлер, розпочав ти,
що їй суджено цвісти й зростати.
Користують із науки графів
математики усіх парафій.

9. То ж, колеги, підіймаймо чаші,
вип'єм дружно за здобутки наші:
графтеоріє, рясній тривало,
любимо тебе й п'ємо во славу!

1. M'lalui Pregel tujuh jembatannya
   Jumlah yang tak dapat diabaikan
   Pemimpin kota yang sangat bangga
   Membangun jembatan menakjubkan

2. Malam hari banyak keramaian
   Jalan jalan di atas jembatan
   Namun pertanyaan masih ada
   Memilih jalan yang paling tepat

3. Tiap jalan harus dilalui
   Tapi jangan kau lupa kembali
   Namun itu tidak kan terjadi
   Satu jembatan tak terlalui

Reff. Eulerian graph, derajatnya genap
Fakta ini kan selalu benar
Teorema tertua
Di dunia graf kita

4. Rakyat kota merasa gelisah
   Semua daya membawa kecewa
   Untungnya ada satu jawara
   Ahli Matematika di sana

5. Tuan Euler dapat menjawabnya
   Dia bilang memang tidak mungkin
   Meniti tiap tujuh jembatan
   S'kali saja dan terus kembali

Reff. Eulerian graph ...

6. Ilmu alam punya aturannya
   Tak satupun dapat mengubahnya
   Jembatan dan badai pun tak bisa
   Mempengaruhi hukum yang ada

7. Saat perang datang menyergapnya
   Jembatannya jatuh berantakan
   Namun nama Euler tetap jaya
   Dikenang untuk s'lama - lamanya

Reff. Eulerian graph ...

8. Nama Euler selalu abadi
   Sepanjang Teori Graf bersemi
   Semakin lama makin berkembang
   Tumbuh bersama aplikasinya

9. Ayo kawan, kita bergembira
   Mari kita saling merayakan
   Kemenangan dan kejayaannya
   Teori graf yang kita cintai.

1. Il y avait sept ponts sur la Pregel
   c'est bien plus qu'on eût pu espérer.
   Les édiles de Koenisberg étaient très fiers
   d'avoir construit de si belles paserelles.

2. Le soir tous allaient vers la rivière,
   parcourant les ponts de long en large,
   se posant constamment la question:
   comment trouver la bonne solution?

3. Ils voulaient traverser chaque pont
   puis s'en retourner à la maison,
   malheureusement cela ne marchait jamais,
   toujours des ponts en trop il y avait.

Ref. Les graphes d'Euler sont de degré pair,
c'est un résultat toujours valide,
c'est le plus vieux théorème
qu'on connait dans ce domaine.

4. A la ville, il y avait un homme sage,
   un vrai savant en mathématiques,
   on pouvait aller le lui demander,
   sûrement aurait-il une bonne idée.

5. Maitre Euler secoua la tête tristement,
   on n'peut passer par chaque pont une seule fois,
   et même si le roi le demandait
   croyez moi, jamais rien n'y ferait.

Ref. Les graphes d'Euler ...

6. La science possède son propre règlement,
   contre lequel nul ne peut aller,
   ni ponts, ni rivieres ni aucun élément
   ne peut briser ce comportement.

7. Quand la guerre souffla sur la Pregel,
   ses tempêtes ont démoli les ponts,
   mais à la rivière le souvenir d'Euler
   survivra dans la postérité.

Ref. Les graphes d'Euler ...

8. Le nom d'Euler est toujours vivant,
   ainsi que la théorie des graphes,
   et plus le temps passe et plus cette théorie
   fleurit, bourgeonne et s'épanouit.

9. Amis, c'est l'heure de remplir notre verre,
   levons-le bien haut à la santé
   pour les siècles à venir, dans la prospérité,
   de notre chère théorie des graphes.

1

Ref.

1 見よpregel川の上に
7つの橋がかかる
ケーニヒスベルグの誇りとなる
素晴らしい橋だろう

２　今夜もすべての橋を
めぐり家路につこう
その街の人々は
ある日悩み始める

３　我らの７つの橋
すべてを一度わたり
元の場所に帰れるか？
だが誰もできなかった

ref　オイラリアングラフ！　橋をすべて
それがグラフのはじまり　　（それはグラフの問題）
すべてが偶数次
はじまりの定理

４　彼らは思い出した
賢者が街に住むと
幾何と計算の達人
彼なら解けるはずだ

５　オイラーは皆の話に
定理を示し答えた
７つの橋をすべて　　　（この街の７つの橋
そんなことは無理だと　　すべては不可能だと）

ref　オイラリアングラフ！　橋をすべて
それがグラフのはじまり
すべてが偶数次
はじまりの定理

６　科学の法則には
誰もがさからえない
この橋もこの川も
科学を変えられない

７　戦火が街を襲い
７つの橋は消え去る
だがオイラーの名前は
今も語り継がれる

ref　オイラリアングラフ！　橋をすべて
それがグラフのはじまり
すべてが偶数次
はじまりの定理

８　オイラーが作り出した
我らのグラフ理論は
流れるときとともに
さらに深まるだろう

９　さあ皆グラスをとれ
ビールをついでまわろう
科学の未来に乾杯
グラフ理論よ永遠に