Nowhere-zero flows on signed series-parallel graphs

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Joint work with **Tomáš Kaiser**

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SIGNED GRAPHS

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graph with $\pm$ signs on edges.

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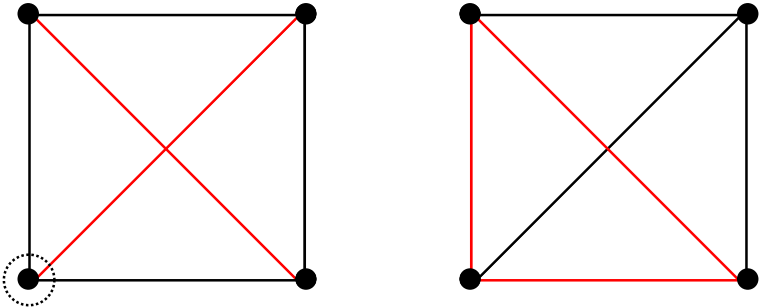

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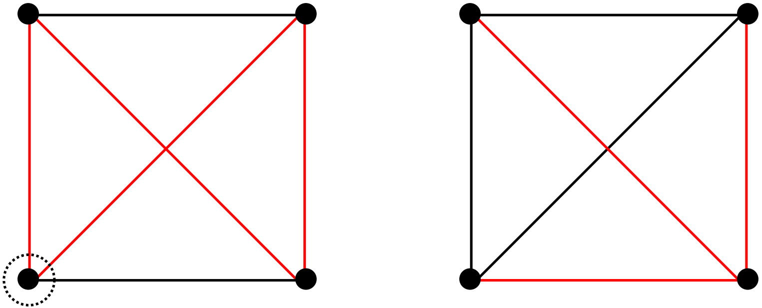
Example 2:



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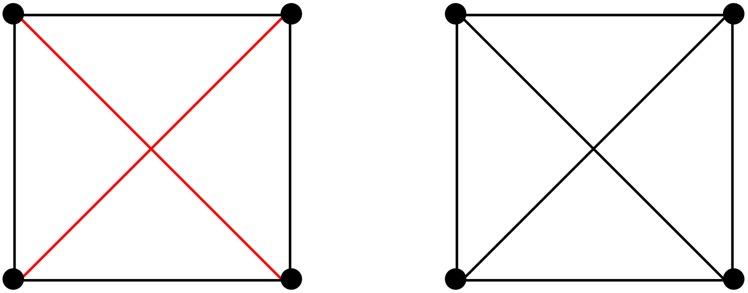

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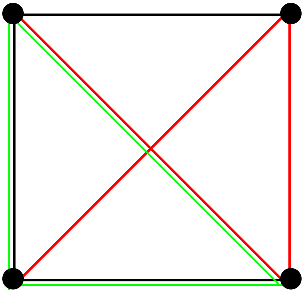
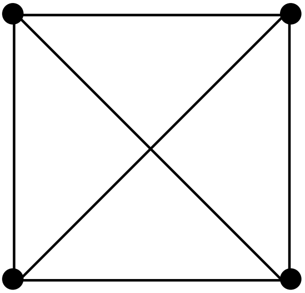
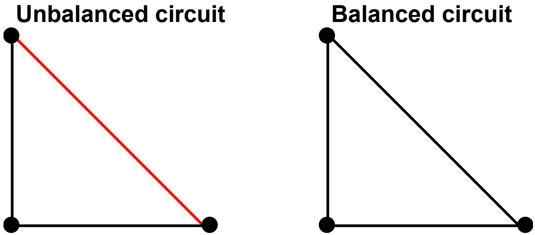




SIGNED GRAPHS

Two kinds of circuits:

- unbalanced (odd number of negative edges)
- balanced (even number of negative edges)



Nowhere-zero (integer) flow:

- orientation of the edges

NOWHERE-ZERO FLOWS ON SIGNED GRAPHS

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- assignment of non-zero integers to edges s.t. for every vertex: sum of incoming values = sum of outgoing values

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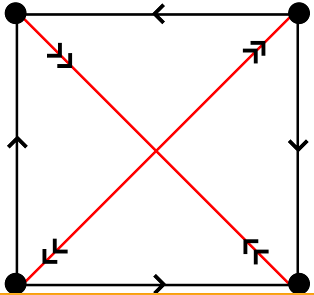
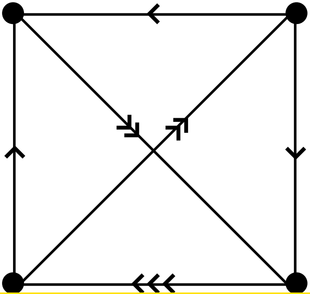
NZ 3-flow

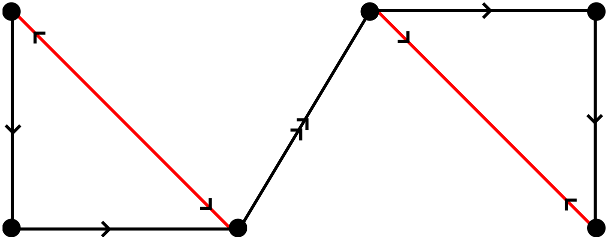


NZ 4-flow

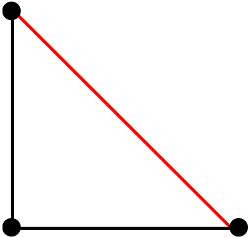


DIFFERENCES

Example of flow-admissible:

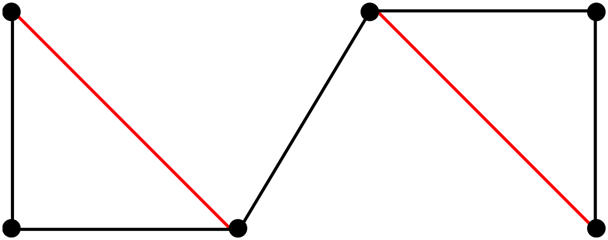


Example of NOT flow-admissible:

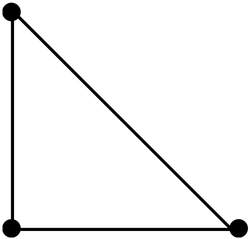


DIFFERENCES

A SIGNED graph is *flow-admissible* if each edge is contained in



or in:

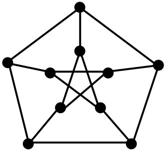


CONJECTURES

Tutte's 5-flow conjecture for (all-positive signed) graphs:

If G admits a NZ flow, then G admits a NZ 5-flow.

(Seymour: 6-flow)



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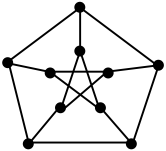
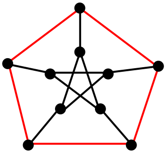


Bouchet's 6-flow conjecture for (general) signed graphs:

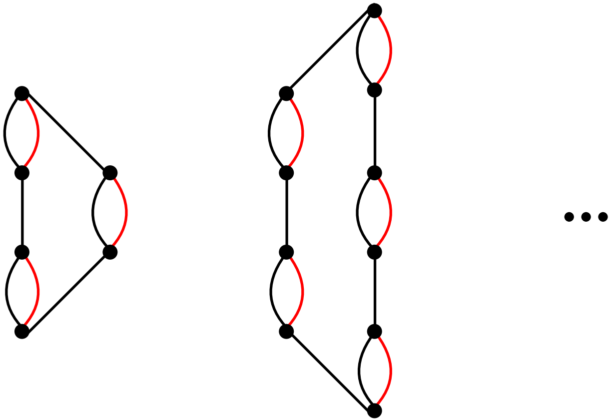
If (G, σ) admits a NZ-flow, then (G, σ) admits a NZ 6-flow.

(Zýka: 30-flow; DeVos: 12-flow)



MOTIVATION

Infinite family of signed graphs with flow number 6
(found by Schubert and Steffen in 2013):



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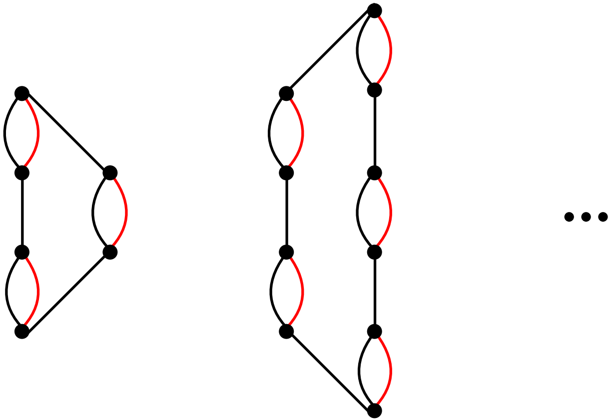


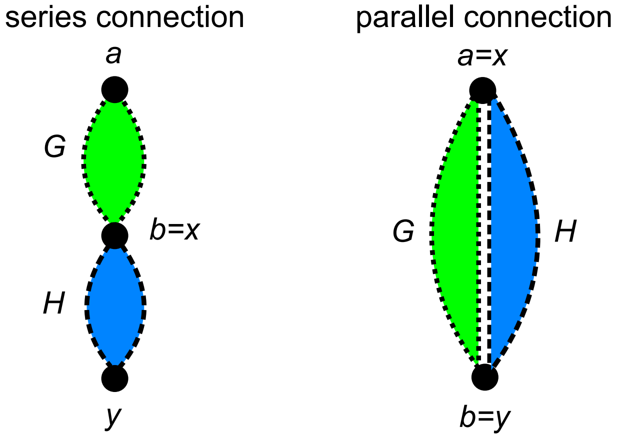
They are signed series-parallel graphs.

SERIES-PARALLEL GRAPHS

A two-terminal series-parallel graph:

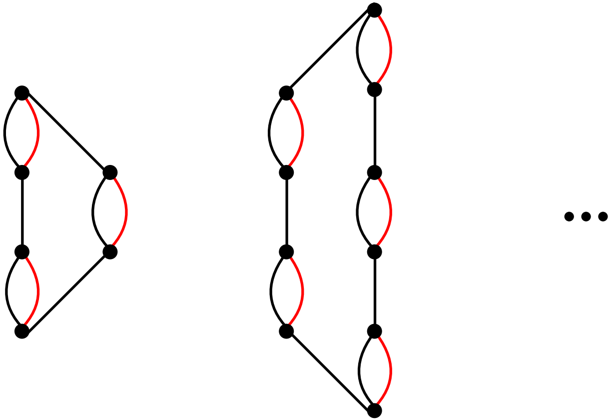
1) $(uv; u, v)$ is series-parallel with terminals u and v

2) If $(G; a, b)$ and $(H; x, y)$ are S-P, then *series connection* and *parallel connection* of G and H are also S-P:



Theorem (Kaiser, R., 2014+)

Every flow-admissible signed series-parallel graph admits a nowhere-zero 6-flow.



Idea of proof.

- reductions

- minimal counterexample
  - is a necklace - it admits a NZ 6-flow
  - contains a necklace

Idea of proof.

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Minimal counterexample does not contain

a) two parallel edges of the same sign
b) non-terminal vertices of degree 2
c) terminal vertices of degree 2 that are not in a 2-cycle
d) balanced end-block if it is a series connection

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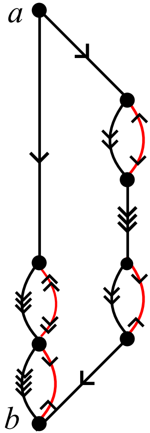

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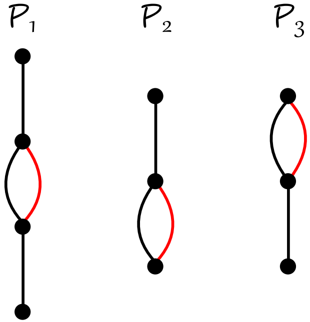

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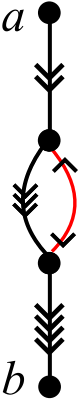

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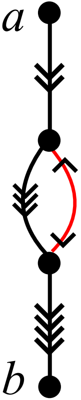

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  5) show that the necklace admits (a, b)-pseudoflow

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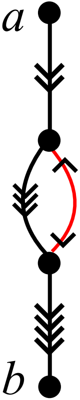

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  4) observe that the piece admits (a, b)-pseudoflow
  5) show that the necklace admits (a, b)-pseudoflow
  6) done!

Thank you for your attention.