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Conference in Algebraic Graph Theory

Symmetry vs. Regularity

The first 50 years since Weisfeiler-Leman stabilization

Book of abstracts

Pilsen, 1 July – 7 July, 2018

ITI

FAKULTA APLIKOVANÝCH VĚD ZÁPADOČESKÉ UNIVERZITY V PLZNI

Symmetry vs Regularity

The main goal of the conference is to reflect recent significant events in the development of Algebraic Graph Theory (AGT) and related areas of mathematics. In particular it will commemorate the 50^{th} anniversary of the discovery of a polynomial-time algorithm for the computation of the coherent closure of a given graph. Over the years, this algorithm has played a significant role in both AGT and computer science.

The abstracts of the talks to be given at the conference boldly demonstrate the accomplishment process of the principal goal of the conference. The talks included in the memorial session trace back the modern Algebraic Graph Theory (AGT) to its founders: I.Schur, R.C.Bose, D.Mesner, D.Higman, B.Weisfeiler, A.Leman, Ph.Delsare, H.Wielandt and others. These talks also mention the famous people contributed to popularization of AGT, notably J.J.Seidel, J.H.Conway and others including some participants of the conference. The keynote and invited lectures cover most of the well established directions, objects and problems of the modern AGT including Association Schemes, Coherent Configurations, Cayley Graphs and Graph Isomorphism Problem. The latter topic is highlighted by the tutorial sessions by L.Babai on his recent discovery of the quasi-polynomial time algorithm. The contributed talks and short presentations further refine the main directions of the current AGT to more specific questions and expose some further applications. It is not unlikely that some of these questions will grow into new important areas of AGT. The variety of themes and objects as well as numerous connections with other mathematical areas demonstrate that currently AGT experiences a very healthy stage of development and expansion. It is hoped by the organizers that the conference will stimulate further these developments and will enable the participants to find new connections and applications of their research.

Scientific committee

Alexander Ivanov, Imperial College, London, United Kingdom (chair)
Mikhail Klin, Ben-Gurion University of the Negev, Be'er Sheva, Israel
Akihiro Munemasa, Tohoku University, Japan
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Logo of WL2018

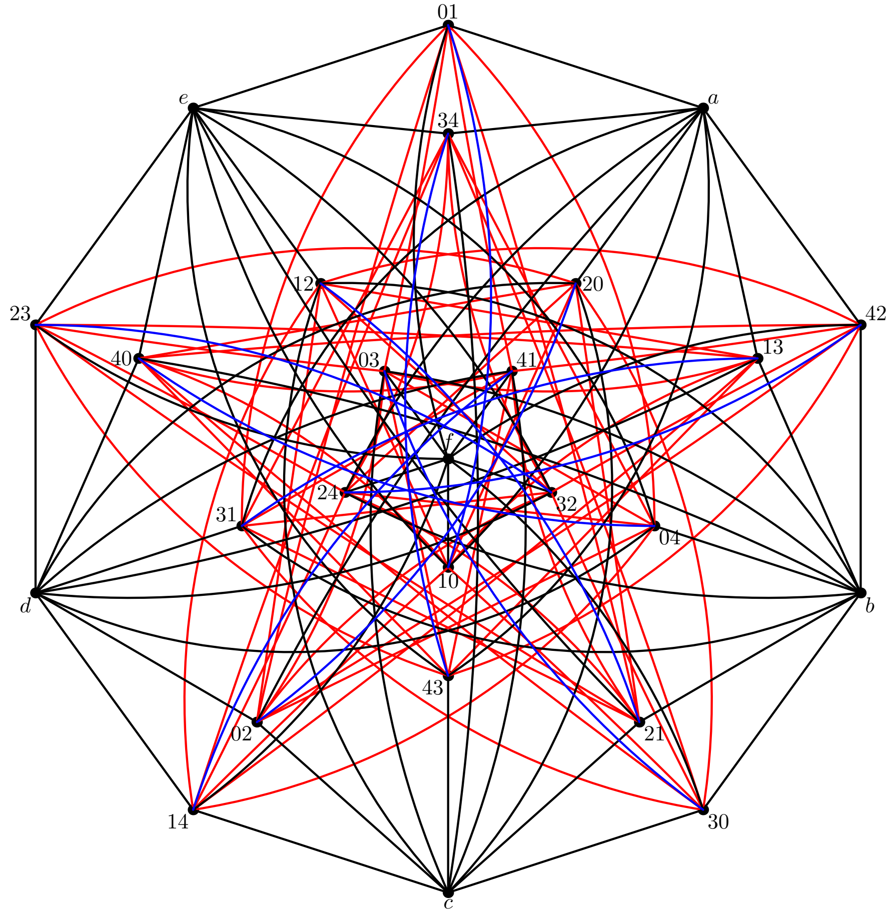
The Paulus-Rozenfeld-Thompson graph T was independently discovered at least three times at Eindhoven (1973), Moscow (1973) and Tucson (1979). It is one of the ten strongly regular graphs (SRGs) with the parameters $(v, k, \lambda, \mu) = (26, 10, 3, 4)$. Among these 10 graphs the SRG T has the largest group $\mathrm{Aut}\, T$ of order 120, which is isomorphic to $A_5 \times Z_2$, the full symmetry group of the dodecahedron Δ, regarded as a distance transitive graph.

In modern terms, T appears as merging of relations of rank 14 coherent configuration W, namely 2-orbits of the induced intransitive action $(A_5, C_1 \cup C_2)$ with two fibers of length 20 and 6. The fiber C_1 consists of all ordered pairs (a, b) from the interval $[0, 4]$. The fiber C_2 consists of the six images of the pentagon $([0, 4], \{\{0, 1\}, \{1, 2\}, \{2, 3\}, \{3, 4\}, \{0, 4\}\})$ under action of $(A_5, [0, 4])$. There are three kinds of edges in the SRG T:

- edges in distance 3, graph Δ_3:
- edges in distance 5, graph Δ_5;
- edge between pair (a, b) and pentagon X, provided $\{a, b\}$ is edge in X.

The diagram presents visual description of the SRG T, as merging of W, on the canvas of Δ with six extra vertices, corresponding to the pentagons with nodes from $[0, 4]$. Different colors correspond to different orbits of A_5 on the edges of T.

A more detailed description of this Logo will appear in article *Animated logo of WL2018.* The logo is created by a team of organizers, consisting of Štefan Gyürki, Mikhail Klin and Matan Ziv-Av.



An alternative drawing of Paulus-Rozenfeld-Thompson graph.

Contents

Programme: Sunday, 1 July

18:00 – 22:00	Registration; informal get together party (Trend)

Programme: Monday, 2 July

08:00 – 09:00	Registration
09:00 – 09:45	Opening
09:50 – 10:50	**Schurian vs non-schurian coherent configurations** Ilia Ponomarenko
10:55 – 11:35	**Coherent configurations with nonsolvable automorphism group** Andrey Vasilyiev
11:35 – 11:50	Coffee break
11:50 – 12:15	**Implementation of stabilization algorithms for configurations** Sven Reichard
12:20 – 12:45	**Graph switching, 2-ranks, and graphical Hadamard matrices** Aida Abiad
12:50 – 13:15	**Finding a maximal subgroup of minimal index in polynomial time** Savelii Skresanov
13:15 – 15:20	Lunch
15:20 – 16:20	**Association schemes, graph homomorphisms, and synchronization** Peter J. Cameron
16:25 – 17:05	**The Graham Higman method and beyond** Alexander Gavrilyuk
17:05 – 17:20	Coffee break

17:20 – 17:45	**Highly regular graphs. Part one: Introduction and examples** Christian Pech
17:50 – 18:15	**Highly regular graphs. Part two: On a family of strongly regular graphs by Brouwer, Ivanov and Klin** Maja Pech
18:45	Dinner
21:30 – 22:10	**Short presentations** (Trend) B. Oliynyk, A. Roca, N. Vanetik, S. Rukavina

Programme: Tuesday, 3 July

08:30 – 09:30	**Isomorphism problem for Cayley objects** Mikhail Muzychuk
09:35 – 10:15	**The Weisfeiler-Leman dimension and the speed of color stabilization — logical and algorithmic aspects** Oleg Verbitsky
10:15 – 10:35	Coffee break
10:35 – 11:00	**On p-valenced association schemes whose thin residue has valency p^2** Mitsugu Hirasaka
11:05 – 11:30	**Commutative Schur rings of maximal dimension** Stephen Humphries
11:35 – 12:00	**Separability and schurity of Cayley schemes over Abelian groups** Grigory Ryabov
12:05 – 12:30	**Discrete version of Fuglede's conjecture and Pompeiu problem** Gábor Somlai
12:30 – 14:30	Lunch

14:30 – 15:30	**Abstract regular polytopes and Y-presentations for sporadic groups** Dmitrii Pasechnik
15:35 – 16:15	**Primitive coherent configurations with very many automorphisms** John Wilmes
16:15 – 16:30	Coffee break
16:30 – 16:55	**Strongly regular graphs constructed from groups** Dean Crnković
17:00 – 17:25	**On packings of disjoint copies of the Hoffman-Singleton graph into K_{50}** Martin Mačaj
18:00	Dinner
20:30 – 22:20	**Short presentations** (Trend) C. Guo, V. Lebid, D. Churikov, I. Estélyi, L. Jörgensen, M. Kagan, M. Koch, D. Kotlar, T. Miyazaki, D. Vagnozzi

Programme: Wednesday, 4 July

08:30 – 09:30	**Applications of semidefinite programming, symmetry, and algebra to graph partitioning problems** Edwin van Dam
09:35 – 10:15	**Integral Cayley graphs on** Sym_n **and** A_n Elena Konstantinova
10:15 – 10:35	Coffee break
10:35 – 11:00	PI**-eigenfunctions of the star graphs** Sergey Goryainov
11:05 – 11:30	**Deza graphs with parameters** $(n, k, k-1, a)$ Vladislav Kabanov

11:35 – 12:00	**On asymptotics and arithmetical properties of complexity for circulant graphs** Alexander Mednykh
12:05 – 12:30	**Geometric structures on the complement of a knotted θ-graph embedded in $\mathbb{S}^3$** Nikolay Abrosimov
12:30 – 14:30	Lunch
14:30	Excursions

Programme: Thursday, 5 July

08:30 – 09:10	**How group theory and statistics met in association schemes** Rosemary Bailey
09:15 – 09:40	**Schur, Wielandt, Tamaschke: The development of S-rings into a tool for group theorists** Kenneth W. Johnson
09:45 – 10:20	**"Symmetry vs Regularity". How it started and what it led to** Igor Faradjev
10:20 – 10:40	Coffee break
10:40 – 11:05	**On the power of WL[k]** Martin Fürer
11:10 – 11:45	**Variations on some themes in the work of Donald G. Higman** Alyssa Sankey
11:50 – 12:20	**Jaap Seidel's network** Willem Haemers
11:50 – 12:20	**Remembering Volodya Zaichenko** Andrei Ivanov
12:45 – 14:45	Lunch

14:45 – 15:35	**History of the study of association schemes, a personal view** Eiichi Bannai
15:40 – 16:15	**Boris Weisfeiler (1941-1985?). Life of mathematician, adventurer, and uncle** Lev Weisfeiler
16:15 – 16:30	Coffee break
16:30 – 16:55	**Dale Mesner and his contributions to algebraic combinatorics** Robert Jajcay
17:00 – 17:15	**Issai Schur, Helmut Wielandt and schurian Schur-rings** Reinhard Pöschel
17:20 – 17:50	**Graphs and groups** Robert Curtis
17:55 – 18:10	Ad hoc comments
20:00	**Conference dinner**

Programme: Friday, 6 July

08:30 – 09:30	**Symmetry vs Regularity** László Babai
09:40 – 10:20	**Minimal degree of the automorphism group of primitive coherent configurations** Bohdan Kivva
10:20 – 10:40	Coffee break
10:40 – 11:40	**Towards the classification of (P and Q)-polynomial schemes** Tatsuro Ito

11:50 – 12:15	**Edge-regular graphs and regular cliques** Gary Greaves
12:20 – 12:45	**Coherent configurations from quantum permutation groups** David Roberson
12:50 – 14:50	Lunch
14:50 – 15:50	**Tutorial: Coherent configurations and the Graph Isomorphism problem I** László Babai
15:50 – 16:10	Coffee break
16:10 – 16:35	**The Weisfeiler-Leman dimension of graphs and isomorphism testing** Pascal Schweitzer
16:35 – 17:00	**Two-fold orbitals** Josef Lauri
17:00 – 17:40	**Special presentation, TBA** Alexander I. Ivanov
18:20	Dinner

Programme: Saturday, 7 July

08:30 – 09:30	**Tutorial: Coherent configurations and the Graph Isomorphism problem II** László Babai
09:40 – 10:05	**The Graph Isomorphism Problem and the Module Isomorphism Problem** Harm Derksen
10:05 – 10:20	Coffee break
10:25 – 11:25	**From transposition groups to algebras** Sergey Shpectorov
11:30 – 11:55	**Constructing Majorana representations** Madeleine Whybrow

12:00 – 12:25	**On pronormal subgroups in finite groups** Natalia Maslova
12:30	Closing the conference
13:00	Lunch

Abstracts

Keynote lectures

Association schemes, graph homomorphisms, and synchronization

Peter J. Cameron
University of St Andrews, Fife, UK

One of the people who developed the theory of coherent configurations was Donald Higman. He came to Oxford in 1970–1971, where (in addition to being my DPhil examiner) he gave a course of lectures on the subject, entitled "Combinatorial Considerations about Permutation Groups"; the notes were taken by Susanna Howard and me, and were published in the Oxford Mathematical Institute Lecture Notes series.

Recently, applications of this topic have emerged in the theory of synchronizing finite automata, where a kind of "duality" between graphs and transformation monoids is important. Investigation of some particular cases, especially the Johnson scheme, led Mohammed Aljohani, John Bamberg and me to a conjecture which would extend Peter Keevash's celebrated result about the existence of Steiner systems.

My talk will reminisce about these topics.

Applications of semidefinite programming, symmetry, and algebra to graph partitioning problems

Edwin van Dam
Tilburg University, Netherlands

A joint work with Renata Sotirov.

We will present semidefinite programming (SDP) and eigenvalue bounds for several graph partitioning problems.

The graph partition problem (GPP) is about partitioning the vertex set of a graph into a given number of sets of given sizes such that the total weight of edges joining different sets — the cut — is optimized. We show how to simplify known SDP relaxations for the GPP for graphs with symmetry so that they can be solved fast, using coherent algebras.

We then consider several SDP relaxations for the max-k-cut problem, which is about partitioning the vertex set into k sets (of arbitrary sizes) such that the cut is maximized. For the solution of the weakest SDP relaxation, we use an algebra built from the Laplacian eigenvalue decomposition — the Laplacian algebra — to obtain a closed form expression that includes the largest Laplacian eigenvalue of the graph. This bound is exploited to derive an eigenvalue bound for the chromatic number of a graph. For regular graphs, the new bound on the chromatic number is the same as the well-known Hoffman bound. We demonstrate the quality of the presented bounds for several families of graphs, such as walk-regular graphs, strongly regular graphs, and graphs from the Hamming association scheme.

If time permits, we will also consider the bandwidth problem for graphs. Using symmetry, SDP, and by relating it to the min-cut problem, we obtain best known bounds for the bandwidth of Hamming, Johnson, and Kneser graphs up to 216 vertices.

Towards the classification of (P and Q)-polynomial schemes

Tatsuro Ito
Anhui University, China

The classification of (P and Q)-polynomial schemes was proposed by Eiichi Bannai in his lectures at Ohio State University late in the 70s. He made a list of known (P and Q)-polynomial schemes, which convinced him that (P and Q)-polynomial schemes are the discrete analogue of compact symmetric spaces of rank 1. He conjectured that (P and Q)-polynomial schemes with sufficiently large diameter are either in his list or some sort of relatives to those in his list.

Starting with the definition of (P and Q)-polynomial schemes and an explanation of the list of Bannai, I will give a survey of the present status of the classification, which falls into two problems: (A) to show (P and Q)-polynomial schemes have the same parameters as in Bannai's list, (B) to characterize the (P and Q)-polynomial schemes in Bannai's list by the parameters. I will also discuss representation theory of Terwilliger algebras of (P and Q)-polynomial schemes in some depth in relation to the problem (A), focusing on the most vital open problems about it.

Isomorphism problem for Cayley objects

Mikhail Muzychuk
Ben-Gurion University of the Negev, Be'er Sheva, Israel

A Cayley object over a finite group H is any relational structure $\mathcal{R}$ with point set H which is invariant under the group of right translations H_R. The well-known examples of Cayley objects include Cayley graphs, Cayley maps, group codes etc. The isomorphism problem for Cayley objects may be formulated as follows:

Given two combinatorial objects over the group H, find whether they are isomorphic or not.

In my talk I'll present the old and the new results which solves the above problem for different classes of objects.

Abstract regular polytopes and Y-presentations for sporadic groups

Dmitrii Pasechnik
University of Oxford, UK

Presentations for sporadic groups by a Y_{abc}-shaped Coxeter diagram with $1 + a + b + c$ generators and few extra relations appear in a number of contexts, cf. e.g. [2, 3, 4, 6, 11, 12]. We observe that an easy "twisting" construction applied to Y_{aba} gives rise to a quotient of a Coxeter group with linear diagram $\{3^{1+a}, 4, 3^{1+b}\}$, and thus (usually) to a combinatorial object called *abstract regular polytope*, studied in the past 40 years, cf. e.g. [1, 5, 7, 8, 10]. This allows to extend and augment the list of abstract *universal* (a.k.a. simply connected) polytopes of type $\{3^2, 4, 3^2\}$, disproving a long-standing conjecture by P. McMullen and E. Schulte [7, 8].

Our new universal polytope is related to a well-known Y-shaped presentation for the sporadic simple group Fi_{22}, and admits $S_4 \times O_8^+(2){:}S_3$ as the automorphism group. We also discuss further extensions of its quotients in the context of Y-shaped presentations. Also, two other known examples of universal $\{3^2, 4, 3^2\}$-polytopes are related to Y-shaped presentations of orthogonal groups over $\mathbb{F}_2$. More details may be found in [9].

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Schurian vs non-schurian coherent configurations

Ilia Ponomarenko
St Petersburg Department of Steklov Institute of Mathematics, Russia

Schurian coherent configurations are those formed by the orbitals of permutation groups. They are exactly the closed objects with respect to the Galois correspondence between coherent configurations and permutation groups. The property of a coherent configuration to be schurian is preserved under taking quotients, direct sums, tensor and wreath products, etc.

The vast majority of coherent configurations are non-schurian. In a sense, this is a reason that the graph isomorphism problem is difficult. Moreover, a non-schurian coherent configuration can be arbitrarily close to a schurian one; this shows that purely combinatorial methods (like the multidimensional Weisfeiler-Leman algorithm) are not sufficient to solve the graph isomorphism problem.

A lot of known results can be formulated in terms of the schurity problem, which consists in identifying schurian coherent configurations in a given class. For example, the Tits theorem on spherical buildings essentially states that the homogeneous coherent configurations associated with the buildings of rank at least three are schurian. Two other recent results, worthy of attention, relate the schurity of circulant schemes and solving linear systems modular congruences, and the schurity of quasiregular coherent configurations with the existence of amalgam of finite abelian groups.

From transposition groups to algebras

Sergey Shpectorov
University of Birmingham, UK

It is well known that the majority of finite simple groups arise as the groups of automorphisms of Lie algebras, which form an important class of non-associative algebras. It is somewhat less known that classical and some exceptional groups are related to Jordan algebras which are another important class of non-associative algebras. Recently this again came into focus due to the introduction of the axial algebra paradigm, pioneered by the Majorana algebras of Ivanov.

In the talk we will discuss these recent developments and present the construction of a class of new algebras with “Monster” fusion.

Memorial session

How group theory and statistics met in association schemes

Rosemary Bailey
University of St Andrews, Fife, UK

This talk has two beginnings, both in the 1930s. I. Schur considered the orbits, of a transitive permutation group, on ordered pairs of points. The partition into orbits is very natural, and properties of the partition are helpful in understanding the groups. This is one fore-runner of coherent configurations. R. C. Bose and his collaborators and students generalized earlier work of F. Yates by introducing partially balanced incomplete-block designs for parameters where no balanced incomplete-block design exists. The condition of partial balance ensures that the relevant matrices can be easily inverted by hand, which was important for data analysis in the pre-computer age. This condition relies on the existence of a (symmetric) association scheme.

In the 1950s, D. Mesner worked on association schemes as a PhD student, later combining with Bose to present what is now called the Bose-Mesner algebra. Not only that, he devised a new type of association scheme, which he called negative Latin square type. He found one on 100 points. In the 1960s Bose made the topic interesting to pure mathematicians by naming strongly regular graphs. These proved fruitful in the search for sporadic simple groups, with the result that D. G. Higman and C. C. Sims rediscovered Mesner's association scheme on 100 points.

Meanwhile, other collaborators of Bose's, including K. R. Nair and J. N. Srivastava, were generalizing association schemes in different ways that now fit within the framework of coherent configurations.

I will conclude by mentioning the series of lectures on coherent configurations that D. G. Higman gave to research students in group theory at Oxford when I was there. It is a shame that some of the people that I mention died before all the connections were understood and acknowledged.

History of the study of association schemes, a personal view

Eiichi Bannai
Shanghai Jiao Tong University, Shanghai, China

I was asked to talk on the history of association schemes. So, I will try to talk on it as much as I could. However, it is too big of a subject, and it is not so easy to give a comprehensive picture. So, please allow me to give a talk only based on my personal view.

It seems that the origin of association schemes, as well as the name, is in statistics, in particular by R.C. Bose and their associates. Another source seems to be in group theory, particularly permutation group theory, as seen in the earlier works of Schur, Wielandt and others. In 1960's these two sources discovered each other and the theory of association schemes became very rich, as is seen in earlier papers on permutation groups by the works of Feit-Higman, D.G. Higman, N.L. Biggs, P. Neumann and many others. I think Delsarte (1973) made it very clear that the association schemes provide an ideal uniform framework to study design theory and coding theory. In fact, association schemes play a key role in unifying individual areas such as algebraic graph theory, algebraic coding theory, algebraic design theory, in the name of algebraic combinatorics. This viewpoint was presented in the book: Algebraic Combinatorics I, by Bannai and Ito published in 1984. There, the influence of the classification problem of finite simple groups was eminent and we put the classification problem of P-and Q-polynomial association schemes as the main target of this research direction. Furthermore, the book: Distance Regular Graphs by Brouwer, Cohen and Neumaier published in 1989, gave a very comprehensive treatment on various broad subjects. The study of association schemes (as well as coherent configurations) and various related topics has been flourishing, including many other viewpoints on association schemes as well as connections with other branches of mathematics and other sciences.

I would like to give a very brief overview of recent research directions in association schemes and related topics, e.g. the works of Terwilliger-Ito and many many others, but this may be presented only very partially.

The book by Bannai-Ito mentioned above was immediately translated into Russian (1987) by a group of researchers in Russia at that time. The encounter with the translators and the Russian school of algebraic combinatorics was very important for us. I also would like to recall our encounters with the Russian mathematicians, including Weisfeiler, Faradjev, A.A. Ivanov, A.V. Ivanov, Shpectorov, Klin, Muzychuk, and many others.

Graphs and groups

Robert Curtis
University of Birmingham, UK

We start with a brief description of a Part III course in Cambridge given by J.A. Todd (of the Todd-Coxeter coset enumeration algorithm) in 1968. In this he described in detail the outer automorphism of the symmetric group Sym(6), used this to construct the Steiner system $\mathrm{S}(5, 6, 12)$, and so obtained the Mathieu group M_{12}. He then used the outer automorphism of M_{12} to construct the Steiner system $\mathrm{S}(5, 8, 24)$ and hence obtained the Mathieu group M_{24}.

I shall indulgently reminisce about those early days following the discovery of the Leech lattice and the Conway groups, and speak briefly about others such as John McKay and Mike Guy who played significant roles. The origin of the so-called Miracle Octad Generator (MOG) will be described and some of its applications will be referred to in the sequel.

Some years later an innocent question from a colleague in connection with arc transitive graphs led me to recognise the connection between the Klein map and M_{24}. I called on W.L Edge in his old people's home near Edinburgh to discuss this and will recount my interesting visit in which he recalled his lecturing tour of Ireland in the days when De Valera was the president.

The Klein-Mathieu connection led to the concept of *symmetric generation* of groups which, in many ways, has its roots in algebraic graph theory. After all, given a graph Γ with group of symmetries N, one can ask if there is a larger group G which contains a generating subset of elements corresponding to the elements of Γ along with a copy of N acting on it. This process will be illustrated by obtaining the Higman-Sims group from the beautiful Hoffman-Singleton graph, and graphs preserved by M_{24} will yield the Conway group $\cdot$O and the largest Janko group J_4.

"Symmetry vs Regularity". How it started and what it led to

Igor Faradjev

Institute for Systems Analysis, Federal Research Center "Computer Science and Control" of Russian Academy of Sciences, Russia

The talk is divided to two parts. In the first part is described the author's vision of the process of origin and development of algebraic combinatorics in 1968–1990 years. In more detail the author dwells on the events, in which either he himself participated, or was a direct onlooker. The second part is devoted to the author's personal relationship with A.Leman and B.Weisfeiler and the atmosphere in which Soviet mathematicians lived and worked.

On the power of WL[k]

Martin Fürer

Pennsylvania State University, PA, USA

A wrong conjecture about the power of the k-dimensional Weisfeiler-Leman algorithm led to the discovery of its basic limitation. WL[k] cannot replace the use of group theory for the graph isomorphism problem restricted to graphs of degree 3, unless k grows proportional to the order of the tested graphs.

On the other hand, WL[2] has strong implications in spectral graph theory. This simple discrete algorithm is sufficiently powerful to replace the numerical approximations in the bounded eigenvalue multiplicity case of the graph isomorphism problem. In fact, WL[2] determines not only the length of the projection of a standard basis vector into any eigenspace, but also the angel between any two such projections.

Jaap Seidel's network

Willem Haemers
Tilburg University, Netherlands

In the seventies of the last century I was a student at the Technical University of Eindhoven under supervision of Jaap Seidel. In these years there was much activity in algebraic graph theory and Jaap Seidel played a central role. Jaap had major contributions to the field, and maybe more importantly, he had a large network of mathematical friends and he often initiated collaboration when he found that the research was related. In the presentation I will recall memories of that periode that illustrate the influence of Jaap Seidel to the developments on "symmetry and regularity".

Remembering Volodya Zaichenko

Andrei Ivanov
Nvidia Corporation, USA

When we remember the departed loved ones, we remember with a gratitude what we owe them and what they did for us. . . What they taught us, or what we ourselves learned from them. And we also remember what they gave us intentionally or accidentally, even sometimes without knowing that it was a real gift for us. About some of such gifts, which I received from Volodya Zaichenko in more than 20 years of our close acquaintance, I want to tell.

Dale Mesner and his contributions to algebraic combinatorics

Robert Jajcay
Comenius University, Bratislava, Slovakia

Bose-Mesner algebras and Kramer-Mesner matrices are probably the two best known contributions of Dale Marsh Mesner in the area of discrete mathematics. Dale's contributions are, however, not limited to these two, and beside his mathematical contributions, it was also his deep humanity and collegiality that made him an important figure to value and remember. We present an eye-witness tribute to Dale's legacy as a mathematician and as a human being.

Schur, Wielandt, Tamaschke: The development of S-rings into a tool for group theorists

Kenneth W. Johnson
Penn State University, Abington, USA

I will discuss the history of how Schur's papers were taken up and used in group theory. Three main methods were used to examine permutation groups with regular subgroups: character theory, S-rings and invariant relations. The theory of B-groups (named after Burnside but not to be confused with what are now called Burnside groups) often involved applications of S-rings. Burnside's original results which motivated Schur's papers used group character theory, and some results by other authors also went along this route. However the group character theory papers seem to have been unusually prone to error, and even the "result" of Burnside which motivated the 1933 paper of Schur had a gap, so Schur's proof was in fact the first. Standard references are Wielandt's book [3] the set of notes [2], and the article by P.M. Neumann in [4]. If time permits I may say something about the use of S-rings in probability (see [1]).

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Issai Schur, Helmut Wielandt and schurian Schur-rings

Reinhard Pöschel
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In 1933 I. Schur introduced (matrix) rings with additional structure in connection with a generalization of a theorem of W. Burnside. These rings later (in an equivalent form) were called Schur-rings (S-rings) by H. Wielandt who developed the theory of S-rings considerably. In particular, H. Wielandt gave a counter-example to the presupposition of I. Schur that each S-ring is the transitivity modul of a permutation group (being transitive and containing a regular subgroup).

In the talk this history is sketched which led also to the notion and investigation of so-called schurian as well as non-schurian S-rings.

Variations on some themes in the work of Donald G. Higman

Alyssa Sankey
University od New Brunswick, Canada

There is a comprehensive account of Don Higman's work in a 2009 paper of Bannai, Griess, Praeger, and Scott which also includes many personal stories. After touching on some of the highlights therein, this talk will focus on work from '88 through '95 on strongly regular designs (of the first and second kind), strongly regular decompositions (with Haemers), imprimitive rank 5 schemes, weights and t-graphs, and uniform schemes (unpublished and later fully developed by van Dam, Martin, and Muzychuk). Generally speaking, this work involves analysis of coherent configurations with small rank, and the problem of classifying imprimitive association schemes. Some themes that arise are methods of producing new coherent configurations from old: fusion, refinement, restriction to – and quotients by – parabolics, and Weisfeiler-Leman stabilization.

Boris Weisfeiler (1941-1985?). Life of mathematician, adventurer, and uncle

Lev Weisfeiler

Boris Weisfeiler was a brilliant mathematician and had worldwide recognition. Born in Moscow in 1941, he was a great student and excelled at math from early years.

Soviet system prevented him from achieving success in Russia and he emigrated to the USA in 1975. There he became a math professor and published a number of scientific papers with very significant results.

Boris was also an avid traveler and experienced hiker with intense curiosity to see new things and to meet new people. Boris went on a hiking vacation to Chile in late 1984 and disappeared there without a trace.

His sister, Olga Weisfeiler, led efforts to find out what happened to him for the next 33 years.

The talk will cover his biography and significant dates/events in his life. To learn more about B. Weisfeiler work and his family effort to find him, please visit `http://boris.weisfeiler.com`.

Invited lectures

The Graham Higman method and beyond

Alexander Gavrilyuk
Pusan National University, South Korea

In the monograph "Permutation groups" [2], Peter Cameron gave a proof, which was due to Graham Higman, that a putative Moore graph of valency 57, i.e., a strongly regular graph with parameters $(3250, 57, 0, 1)$, cannot be vertex-transitive. The proof is based on calculating character values of certain representations of the automorphism group of an association scheme and the fact that these values must be algebraic integers. This gives a necessary condition which may rule out the existence of some automorphisms, and we will refer to it as the Higman method. The idea itself probably goes back to the celebrated work of Feit and Higman on generalized polygons [4], which inspired Benson [1] to apply it to generalized quadrangles with prescribed symmetries. (The result of Benson was extended to other types of incidence structures and became known in finite geometry as the Benson type theorems, [3].)

In this survey talk, we give an overview of the Higman (-Benson) method and its recent applications [3, 5, 6, 7] which include characterizations, constructions and non-existence results of some strongly/distance regular graphs with prescribed symmetries.

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Minimal degree of the automorphism group of primitive coherent configurations

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The minimal degree of a permutation group G is the minimum number of points not fixed by non-identity elements of G. Lower bounds on the minimal degree have strong structural consequences on G. In 2014 Babai proved that the automorphism group of a strongly regular graph with n vertices has minimal degree at least cn, with known exceptions. Strongly regular graphs correspond to primitive coherent configurations of rank 3. We extend Babai's result to primitive coherent configurations of rank 4. We also show that the result extends to non-geometric primitive distance-regular graphs of bounded diameter. The proofs combine structural and spectral methods. The results have consequences to primitive permutation groups that were previously known using the classification of finite simple groups (Cameron, Liebeck).

Integral Cayley graphs on Sym_n and A_n

Elena Konstantinova
Sobolev Institute of Mathematics, Novosibirsk State University, Russia

The problem of classifying integral graphs was suggested by F. Harary and A.J. Schwenk in the 1970's. A graph is said to be integral if all eigenvalues of the adjacency matrix are integers. Since the general problem of classifying integral graphs seems too difficult, special classes of graphs are investigated. In this talk we discuss recent progress on characterizing integral Cayley graphs on the symmetric group Sym_n and the alternating group A_n.

Coherent configurations with nonsolvable automorphism group

Andrey Vasilyiev
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Let $\mathcal{X} = (\Omega, S)$ be a coherent configuration on a set Ω. Due to the well-known Babai–Lucks result there is a polynomial-time algorithm which finds the automorphisms of $\mathcal{X}$ inside any given permutation group $H \leq \mathrm{Sym}(\Omega)$ of composition width bounded by a constant. This explains an interest to coherent configurations having nonsolvable automorphism group. One can get such a configuration $\mathcal{X}$ by taking the orbitals of a nonsolvable group G. In this case, G is a subgroup of the automorphism group $\mathrm{Aut}(\mathcal{X})$ of $\mathcal{X}$, and if G is nonsolvable, so is $\mathrm{Aut}(\mathcal{X})$. The same picture arises in the case of Cayley graphs (or Cayley schemes) over nonsolvable groups. Indeed, if Γ is a Cayley graph for a (nonsolvable) group G, then G is included in $\mathrm{Aut}(\Gamma)$ as a regular subgroup. We are going to discuss some new results and techniques on the isomorphism problem for coherent configurations of the above types.

The Weisfeiler-Leman dimension and the speed of color stabilization — logical and algorithmic aspects

Oleg Verbitsky
Humboldt University of Berlin, Germany

Let *WL-dim*(G) denote the *Weisfeiler-Leman dimension* of a graph G, that is, the minimum k such that the k-dimensional Weisfeiler-Leman algorithm identifies G up to isomorphism. Furthermore, let $WL_k(G)$ denote the number of color refinement rounds performed by the k-dimensional Weisfeiler-Leman algorithm on the input G until the partition of $V(G)^k$ stabilizes.

If $WL\text{-}dim(G)$ is bounded by a constant k for all G in a class of graphs $\mathcal{C}$, then the isomorphism problem for $\mathcal{C}$ is solvable in polynomial time, namely in time $O(n^{k+1} \log n)$, where n denotes the number of vertices in G. If, moreover, $WL_k(G) = O(\log n)$ for all $G \in \mathcal{C}$, then the problem is solvable even in logarithmic parallel time.

It is known that $WL\text{-}dim(G) \leq k$ if and only if the graph G is definable up to isomorphism by a first-order sentence using at most $k+1$ variables (possibly with many occurrences) and counting quantifiers. Moreover, the quantifier depth of this sentence is closely related to $WL_k(G)$. In this way, a very efficient isomorphism algorithm for a class of graphs $\mathcal{C}$ can be obtained by proving that every graph in $\mathcal{C}$ is definable in a finite-variable counting logic with logarithmic quantifier depth. This approach works for graphs of bounded tree-width, planar graphs, and interval graphs; see, e.g. [1].

After surveying this line of research, we will consider more general combinatorial questions. Let $\mathcal{C}_k$ denote the class of all graphs G with $WL\text{-}dim(G) \leq k$. Thus, $\mathcal{C}_1$ consists of the graphs whose isomorphism type is identifiable by Color Refinement, and $\mathcal{C}_2$ consists of the graphs identifiable by the original Weisfeiler-Leman algorithm. The class $\mathcal{C}_1$ admits an efficient characterization. Obtaining such a characterization for $\mathcal{C}_2$ is currently out of reach; this would include understanding of which strongly regular graphs are determined by their parameters uniquely.

We will conclude with a discussion of the speed of the Weisfeiler-Leman stabilization. Trivially, $WL_k(G) < n^k$. There are graphs with $WL_1(G) = (1 - o(1))n$, which turns out to be meaningful in the context of distributed computing [2]. On the other hand, it is known [3] that $WL_2(G) = o(n^2)$ for all G.

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Primitive coherent configurations with very many automorphisms

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Coherent configurations (CCs) are highly regular colorings of the set of ordered pairs of a "vertex set"; each color represents a "constituent digraph." Their history goes back to Schur in the 1930s. A CC is primitive (PCC) if all its constituent digraphs are connected.

We address the problem of classifying PCCs with large automorphism groups. This project was started in Babai's 1981 paper [1] in which he showed that only the trivial PCC admits more than $\exp(\widetilde{O}(n^{1/2}))$ automorphisms. (Here, n is the number of vertices and the $\widetilde{O}$ hides polylogarithmic factors.)

In joint work with Xiaorui Sun, we classify all PCCs that have more than $\exp(\widetilde{O}(n^{1/3}))$ automorphisms, making the first progress on Babai's conjectured classification of all PCCs with more than $\exp(n^{\epsilon})$ automorphisms. Specifically, we prove that the only primitive coherent configurations with more than $\exp(\widetilde{O}(n^{1/3}))$ automorphisms are trivial configurations, Johnson schemes, and Hamming schemes.

Our result implies an $\exp(\widetilde{O}(n^{1/3}))$ bound on the order of primitive but not doubly transitive permutation groups, with known exceptions. This result was previously known only through the Classification of Finite Simple Groups [2], while our proof is elementary and almost completely combinatorial.

A crucial element of our proof is the discovery of "asymptotically uniform clique geometries" on PCCs in a certain range of the parameters. In cases when such a geometry is present and has only two cliques at each vertex, we can directly classify all possible PCCs. When such geometries are not present, or when there are more than three cliques at each vertex, we instead find a set of vertices such that individualization of these vertices gives a discrete coloring after two rounds of Weisfeiler-Leman color refinement.

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Contributed talks

Graph switching, 2-ranks, and graphical Hadamard matrices

Aida Abiad
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We study the behaviour of the 2-rank of the adjacency matrix of a graph under Godsil-McKay switching, and apply the result to certain graphs coming from graphical Hadamard matrices of order 4^m. Starting with graphs from known Hadamard matrices of order 64, we find (by computer) many Godsil-McKay switching sets that increase the 2-rank. Thus we find strongly regular graphs with parameters $(63, 32, 16, 16)$, $(64, 36, 20, 20)$, and $(64, 28, 12, 12)$ for almost all feasible 2-ranks. In addition we work out the behaviour of the 2-rank for a graph product related to the Kronecker product for Hadamard matrices, which enables us to find many graphical Hadamard matrices of order 4^m for which the related strongly regular graphs have an unbounded number of different 2-ranks.

This is joint work with S. Butler and W.H. Haemers.

Geometric structures on the complement of a knotted θ-graph embedded in $\mathbb{S}^3$

Nikolay Abrosimov
Laboratory of Topology and Dynamics of Novosibirsk State University, Russia

A joint work with Alexander Mednykh and Daria Sokolova.

We consider the figure-eight knot 4_1 with a bridge which is a knotted θ-graph embedded in $\mathbb{S}^3$. An Euclidean structure on the figure-eight knot arises when its conical angle α equals $2\pi/3$ [1]. An explicit construction of fundamental set for a cone-manifold $4_1(\alpha)$ in E^3 was given in [2]. The existence of the euclidean structure on figure-eight with a bridge was shown in [3].

In the present work we consider a two-parameter family of cone manifolds $4_1(\alpha, \gamma)$ whose singular set is the figure-eight knot with a bridge and conical angles α and γ along them. For such cone manifolds we construct a fundamental set using the special representations of the fundamental group in $PSO(1,3)$ and $PSL(2,C)$. That is a non-convex polyhedron P having 20 triangular faces and 12 vertices embedded into the Cayley-Klein model of H^3. We establish existence conditions for the hyperbolic structure on $4_1(\alpha,\gamma)$.

Theorem 1. *A hyperbolic structure on $4_1(\alpha,\gamma)$ is exist if and only if*

$$\begin{cases} -1+3\,M^2+12\,X^2-4\,M^2X^2-16\,X^4 & \geq 0, \quad (i) \\ 5+6\,M^2+M^4-60\,X^2-12\,M^2X^2+80\,X^4 & > 0, \quad (ii) \end{cases}$$

where $M=\cot\frac{\alpha}{2}, \alpha\in(\frac{\pi}{3},\pi), X=\cos\frac{\theta}{2}, \theta\in(0,\frac{\pi}{2})$ and θ is the angle of relative rotation between singular components. The equality in (i) is achieved under the condition $\gamma=2\pi$, i.e. when the bridge disappears. The equality in (ii) is achieved if there exist an Euclidean structure on $4_1(\alpha,\gamma)$.

Theorem 2. *If cone-manifold $4_1(\alpha,\gamma)$ admits a hyperbolic structure then*

$$\begin{aligned} -\cos\frac{\gamma}{2} &= 8\,u^2-16\,u^4+5\,w-40\,u^2w+80\,u^4w+32\,u^2w^2-128\,u^4w^2 \\ &\quad -20\,w^3+64\,u^2w^3+64\,u^4w^3-64\,u^2w^4+16\,w^5, \end{aligned}$$

where $u=\frac{1}{2}\,\mathrm{tr}A=\frac{1}{2}\,\mathrm{tr}B=\cos\alpha,\ w=\mathrm{tr}(A\,B^{-1})=u^2-(1-u^2)\,\mathrm{ch}\rho$ and $\rho=2\,h+i\,\theta$ is the complex hyperbolic distance between the singular components of $4_1(\alpha,\gamma)$.

This work was supported by the Laboratory of Topology and Dynamics, Novosibirsk State University (contract no. 14.Y26.31.0025 with the Ministry of Education and Science of the Russian Federation).

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Strongly regular graphs constructed from groups

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A construction of 1-designs determined by a transitive action of a finite group is given in [1]. Under certain conditions the incidence matrix of a constructed 1-design is symmetric with zero diagonal, i.e. this matrix is the adjacency matrix of a regular graph. Using this method we construct a number of strongly regular graphs, including the first known examples of strongly regular graphs with parameters $(216, 40, 4, 8)$ and $(540, 187, 58, 68)$. This construction also leads to a construction of some distance-regular graphs with diameter greater than 2. Further, we discuss linear codes obtained from the constructed graphs. The obtained codes usually have large automorphism groups, hence they are suitable for permutation decoding.

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The Graph Isomorphism Problem and the Module Isomorphism Problem

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Brooksbank and Luks gave a polynomial time algorithm for testing whether two given n-dimensional modules are isomorphic. In my talk I will explain how to use this result to build a stronger version of the Weisfeiler-Leman algorithm. This new algorithm can also distinguish pairs of graphs constructed by Cai, Fürer and Immerman in polynomial time that cannot be distinguished in polynomial time by the classical Weisfeiler-Leman method.

PI-eigenfunctions of the star graphs

Sergey Goryainov
Shanghai Jiao Tong University, China

Based on joint work in progress with
Vladislav Kabanov, Elena Konstantinova, Leonid Shalaginov and Alexandr Valyuzhenich.

Denote by Sym_n the symmetric group on $\{1, 2, \ldots, n\}$. We investigate eigenfunctions of the Star graph $S_n = Cay(\mathrm{Sym}_n, S)$, $n \geqslant 2$, which is the Cayley graph on Sym_n with the generating set $S = \{(1\ i) \mid 2 \leqslant i \leqslant n\}$. For any $n \geqslant 4$, the spectrum of the Star graph S_n is integral and consists of all integers in the range $-(n-1), \ldots, n-1$ (see [1]). This follows from the fact that the adjacency matrix of S_n coincides with the transformation matrix of the Jucys-Murphy element $J_n = (1\ n) + \ldots + (n-1\ n)$ acting on the group algebra $\mathbb{C}[\mathrm{Sym}_n]$.

In this talk, for any positive integers $n \geqslant 3$ and m with $n > 2m$, we present a family of $(1, -1, 0)$-eigenfunctions (we call them *PI-eigenfunctions*) of the Star graph S_n with eigenvalue $n - m - 1$, and establish a connection between these eigenfunctions and the standard basis of a Specht module. More precisely, we embed a permutation module into $\mathbb{C}[\mathrm{Sym}_n]$ and prove that an eigenfunction of the Jucys-Murphy operator J_n with eigenvalue $n - m - 1$, $n > 2m$, given by a polytabloid can be expressed as a sum of PI-eigenfunctions of S_n. The discussed results are presented in [2].

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Edge-regular graphs and regular cliques

Gary Greaves
Nanyang Technological University, Singapore

We solve a problem of Neumaier about the existence of non-strongly-regular edge-regular graphs that pos sess regular cliques. In this talk, we will present a construction of such graphs. This talk is based on joint work with Jack Koolen.

On p-valenced association schemes whose thin residue has valency p^2

Mitsugu Hirasaka
Pusan National University, South Korea

Let (X, S) be an association scheme where X is a finite set and S is a partition of $X \times X$, and p a prime. We say that (X, S) is p-valenced if each valency is a power of p. In this talk we focus on p-valenced association schemes with their thin residues of valency p^2. The structure of the thin residue of such association schemes is isomorphic to one of the following: (i) C_{p^2}; (ii) $C_p \times C_p$ or (iii) $C_p \wr C_p$ where C_n means the cyclic group of order n. We aim to summarize known results on (i) and (ii) to show a construction such association schemes of type (iii) by using generalized Hadamard matrices. This is a joint work with J.R. Cho and K. Kim.

Commutative Schur rings of maximal dimension

Stephen Humphries
Brigham Young University, Provo, UT, USA

Let $d_1, \ldots, d_k$ be the degrees of the irreducible representations of a finite group G. Then the dimension of a maximal commutative Schur ring over G is $d_1 + \cdots + d_k$. We determine classes of groups that achieve this maximal dimension. This includes $PSL(2, 2^n)$, metacyclic group, extra special groups, and groups whose character degrees are 1 and p for a fixed prime p. We also determine families of groups that do not attain this bound. We show that the class of groups attaining this bound is invariant under quotients and certain facts about random walks.

Deza graphs with parameters $(n, k, k-1, a)$

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A nonempty k-regular graph G on n vertices is called a Deza graph if there exist constants b and a, $(b \geq a)$ such that any pair of distinct vertices of G has precisely either b or a common neighbours. The quantities n, k, b, and a are called the parameters of G and are written as the quadruple (n, k, b, a). If a Deza graph has diameter 2 and is not strongly regular, then it is called a strictly Deza graph. We classified all strictly Deza graphs with parameters $(n, k, k-1, a)$. This is a joint work with S. Goryainov, N. Maslova, L. Shalaginov

Two-fold orbitals

Josef Lauri
Josef Lauri, University of Malta, Malta

Let V be a finite set and let G be a subgroup of $S_V \times S_V$, where S_V is the symmetric group of all permutations on the set V. We define the action of G on $V \times V$ in the following natural manner: if $(f, g) \in G$ and $(u, v) \in V \times V$ then $(u, v)^{(f,g)}$ is defined to be (u^f, v^g). Such a permutation of $V \times V$ is said to be a *two-fold permutation* of $V \times V$.

I shall very briefly show how two-fold permutations enabled us to construct a smallest possible unstable graph with trivial automorphism group and also a family of asymmetric graphs with an arbitrarily high index of instability. (A graph Γ is said to be *stable* if $|\mathrm{Aut}(\Gamma)| = 2|\mathrm{Aut}(\Gamma \times K_2)|$, where $\times$ here denotes the direct product of graphs; also, the *index of instability* of Γ is defined to be $|\mathrm{Aut}(\Gamma \times K_2)|/2|\mathrm{Aut}(\Gamma)|$.)

But the best context in which to study two-fold permutations is to consider the family of orbits of the action of G on the ordered pairs $V \times V$, which is somewhat analogous to the study of 2-orbits of a permutation group acting on the set V, and which we therefore call *two-fold orbitals*. But unlike orbitals, a family of two-fold orbitals can never form a coherent configuration since some orbits must contain loops together with ordered pairs of distinct elements of V. However, under certain conditions, a family of two-fold orbitals does satisfy relationships which give a well-definition of structure constants analogous to those defined for coherent configurations. I shall discuss these conditions and also show the results of some experimentation we carried out on two-fold orbitals using `COCO-II` and `GAP`.

On packings of disjoint copies of the Hoffman-Singleton graph into K_{50}

Martin Mačaj
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In 1983, A.J. Schwenk asked whether it is possible to decompose K_{10} into three copies of the Petersen graph. In an analogous way, we may ask whether there exists a decomposition of K_{50} into seven copies of the Hoffman-Singleton graph (HoSi).

Further, in a paper published in 2010, E.R. van Dam and M. Muzychuk asked whether there exists a $SRG(50, 21, 8, 9)$ which can be decomposed into three copies of HoSi. Similarly, we may ask for a decomposition of a $SRG(50, 28, 15, 16)$ into four copies of HoSi.

Using methods of J. Šiagiová and M. Meszka, we answer these questions under the condition that all HoSis share a non-trivial automorphism.

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On pronormal subgroups in finite groups

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This talk is based on joint papers with W. Guo, A. S. Kondrat́ev, and D. O. Revin.

According to Ph. Hall, a subgroup H of a group G is said to be *pronormal* in G if H and H^g are conjugate in $\langle H, H^g \rangle$ for every $g \in G$. Obvious examples of pronormal subgroups are normal subgroups, maximal subgroups, and Sylow subgroups of finite groups; Sylow subgroups of proper normal subgroups of finite groups; Hall subgroups of finite solvable groups.

With respect to a result by Ph. Hall [Theorem 6.6, 4], a subgroup H is pronormal in a finite group G if and only if in any transitive permutation representation of G, the subgroup $N_G(H)$ acts transitively on the set $fix(H)$. Thus, the pronormality of a subgroup is very closely connected with properties of permutation representations of a finite group. Moreover, pronormality is the universal property with respect to the Frattini Argument (see [11] and [Lemma 4, 4].

A number of problems in combinatorics and permutation group theory were solved in terms of the pronormality (see, for example [1,9,10]). The following problem naturally arises.

Problem. Given a finite group G and its subgroup H. We wish to be able to answer the following question: Is H pronormal in G?

Assume that G is not simple, and A is a minimal normal subgroup of G. In all the cases, except when A is nonabelian and $G = HA$, the solution of Problem could be reduced to solutions of Problem for groups of order less than the order of G. But the case when A is nonabelian and $G = HA$ is really very difficult. So, it is necessary to obtain some nice restrictions to G and H.

With respect to a result by Ch. Praeger [10], if G is a transitive permutation group on a set of n points and K is a nontrivial pronormal subgroup of G, then $|fix(K)| \leq \frac{1}{2}(n-1)$; moreover, if $|fix(K)| = \frac{1}{2}(n-1)$, then G and K are known. Thus, it is important to consider pronormality question of overgroups of a-priori pronormal subgroups, in particular, of Sylow subgroups.

In [Theorem 4, 2]we obtained a criteria of pronormality of overgroups of Sylow p-subgroups in finite groups G such that G contains a nontrivial normal subgroup A, and the overgroups of Sylow p-subgroups are pronormal in A. Thus, we are interested on pronormality question for overgroups of Sylow subgroups in finite nonabelian simple groups and in direct products of finite nonabelian simple groups. Here the situations for Sylow subgroups of odd order and for Sylow 2-subgroups are really distinct.

The following proposition is a nice corollary of Ph. Hall's result [Theorem 6.6, 4]. Suppose that G is a finite group, $H \leq G$, and H contains a Sylow p-subgroup S of G. Then H is pronormal in G if and only if the subgroups H and H^g are conjugate in $\langle H, H^g \rangle$ for every $g \in N_G(S)$. Thus, if Sylow p-subgroups are self-normalized in a finite group G, then the overgroups of Sylow p-subgroups are pronormal in G. Now, if $p = 2$, then the Sylow 2-subgroups are often self-normalized in finite simple groups (see [5]). However, if p is odd, then the Sylow p-subgroups are never self-normalized in finite simple groups (see [3]).

In [12], Vdovin and Revin conjectured that the subgroups of odd index are pronormal in finite simple groups. The conjecture was verified for many families of finite simple groups in [6]. Namely, it was proved that the subgroups of odd index are pronormal in the following finite simple groups: A_n, where $n \geq 5$; sporadic groups; groups of Lie type over fields of characteristic 2; $PSL_{2^n}(q)$; $PSU_{2^n}(q)$; $PSp_{2n}(q)$, where $q \not\equiv \pm 3 \pmod 8$; $P\Omega_{2n+1}(q)$; $P\Omega^{\varepsilon}_{2n}(q)$, where $\varepsilon \in \{+, -\}$; exceptional groups of Lie type not isomorphic to $E_6(q)$ or ${}^2E_6(q)$.

In [7, 8] it was proved that the conjecture fails. Precisely, if $q \equiv \pm 3 \pmod 8$ and $n \notin \{2^m, 2^m(2^{2k}+1) \mid m, k \in \mathbb{N} \cup \{0\}\}$, then the finite simple symplectic group $PSp_{2n}(q)$ contains a non-pronormal subgroup of odd index. In this talk we discuss a pronormality question for subgroups of odd index in finite groups. In particular, we discuss a recent progress in the classification of finite simple groups in which the subgroups of odd index are pronormal, and some corresponding results for direct products of finite simple groups.

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On asymptotics and arithmetical properties of complexity for circulant graphs

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We study analytical and arithmetical properties of the complexity function for infinite families of circulant graphs $C_n(s_1, s_2, \ldots, s_k)$ and $C_{2n}(s_1, s_2, \ldots, s_k, n)$. Exact analytical formulas for the complexity functions of these families are derived, and their asymptotics are found. As a consequence, we show that the thermodynamic limit of these families of graphs coincides with the small Mahler measure of the accompanying Laurent polynomials.

Highly regular graphs. Part one: Introduction and examples

Christian Pech

A *regularity type* $\mathbb{T}$ of order (m, n) is a triple (Δ, ι, Θ), where Δ and Θ are graphs of order m and n, respectively, and where $\iota\colon \Delta \hookrightarrow \Theta$ is an embedding. A graph Γ is called $\mathbb{T}$-*regular* if for all $\kappa\colon \Delta \hookrightarrow \Gamma$ the number of embeddings $\hat{\kappa}\colon \Theta \hookrightarrow \Gamma$ with $\kappa = \hat{\kappa} \circ \iota$ is equal to a constant $\#(\Gamma, \mathbb{T})$ (i.e., it does not depend on κ). A graph is called (m, n)-regular if it is $\mathbb{T}$-regular, for all regularity types $\mathbb{T}$ of order (k, l), for $k \leq m$, $l \leq n$. Note that strongly regular graphs are just the $(2, 3)$-regular graphs and graphs that satisfy the t-vertex condition correspond to the $(2, t)$-regular graphs. Moreover, the k-isoregular graphs are just the $(k, k+1)$-regular graphs. Often, regularity is entailed by symmetry. Recall that a graph is called m-homogeneous if every isomorphism between induced subgraphs of order $\leq n$ extends to an automorphism of the graph. Clearly, if a graph is m-homogeneous, then it is (m, n) regular, for all $n \geq m$. We are interested into regularities that are not entailed by symmetries. We call a graph Γ *highly regular* if there is some $m \geq 2$ and some $n \geq 4$, such that Γ is (m, n)-regular, but not m-homogeneous. Note that high regularity does not completely exclude high symmetry. The only request that we have from a highly regular graph is that there exists some degree of regularity that is not explainable by symmetries. E.g., the McLaughlin graph on 275 vertices is $(4, 5)$-regular but is not 4-homogeneous. So in particular it is highly regular. On the other hand it is 3-homogeneous. In the first part of our presentation we introduce the theoretical framework of regularity conditions and we give an overview of known highly regular graphs.

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Highly regular graphs. Part two: On a family of strongly regular graphs by Brouwer, Ivanov and Klin

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In 1989 A.V.Ivanov discovered a $(2,5)$-regular graph whose subconstituents are $(2,4)$-regular. His construction was generalized by Brouwer, Ivanov and Klin to an infinite family of strongly regular graphs. They showed that all members of the family are $(3,4)$-regular and that their first subconstituents are $(2,4)$-regular but not 2-homogeneous. Only much later on it was shown by Reichard that these graphs are also $(2,5)$-regular. In this talk we are going to reanalyze the Brouwer-Ivanov-Klin graphs using the techniques introduced in the first part thus uncovering regularities of these graphs that hitherto remained hidden.

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Implementation of stabilization algorithms for configurations

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One origin of the notion of coherent configurations is the study of the complexity of the graph isomorphism problem. Given an edge-coloring of a complete graph the algorithm by Weisfeiler and Leman [8] constructs a refinement of the coloring using invariants on the edges; the resulting refined coloring has the same automorphisms as the original coloring. The invariant that is used counts colored triangles involving a given edge. Coherent configurations may be defined as colorings that are stable under this procedure.

The concepts above can be generalized in several directions. Instead of graphs we can consider (uniform) hypergraphs; this leads to k-ary coherent configurations [6], which play a prominent role in recent advances in the study of the graph isomorphism problem [1]. In this k-dimensional WL algorithm one uses structures of order $k+1$ containing a given k-tuple. Another invariant for graphs is related to the *t-vertex condition* [5]; here one looks at t-vertex subgraphs containing a given pair of vertices. We look at a common generalization of both directions, considering structures of arbitrary order t containing a given k-tuple.

It is known that the algorithm of Weisfeiler-Leman is polynomial in complexity [3, 7]. Twenty years ago two implementations were described [2] which both had their advantages and disadvantages.

We will present a new implementation of a more general framework that takes into account the generalizations described above as well as advances of modern computer architecture. Main features include methods for the quick evaluation of invariants as well as parallelization [4]. A practical demonstration will be given; the programs will be available as open source.

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Coherent configurations from quantum permutation groups

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Given a permutation group G acting on a set X, the orbits of G on $X \times X$ (often called the "orbitals" of G) form a coherent configuration. In a recent work, we have shown how to define orbitals for *quantum* permutation groups, and moreover have shown that these also form a coherent configuration. This allows us to apply techniques from the study of graph isomorphism to the study of the recently defined notion of quantum isomorphism of graphs. Given a graph X, its *quantum* automorphism group gives rise to a coherent configuration on $V(X)$ as described above. We show that for quantum isomorphic graphs X and Y, the coherent configurations arising from their quantum automorphism groups must be (weakly) isomorphic. This further implies that the graphs are not distinguished by the Weisfeiler-Leman algorithm, thus providing an efficiently computable necessary condition for two graphs to be quantum isomorphic.

Joint work with Martino Lupini and Laura Mancinska. Based on `arXiv:1712.01820`.

Separability and schurity of Cayley schemes over Abelian groups

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A coherent configuration is called *separable* if every algebraic isomorphism from it to another coherent configuration is induced by a combinatorial one. A finite group G is said to be *separable* if every Cayley scheme over G is separable. In the case when a group G is separable an isomorphism of two given Cayley graphs over G can be tested by using the Weisfeiler-Leman algorithm. Indeed, given two graphs this algorithm enables to check whether there exists an algebraic isomorphism of the corresponding coherent configurations that maps the arc set of the first graph to that of the second.

It was proved in [1] that all cyclic p-groups are separable. One can prove that a noncyclic abelian separable p-group is isomorphic to $C_p \times C_{p^k}$ or $C_p \times C_p \times C_{p^k}$, where $p \leq 3$ and $k \geq 1$. We prove the following statement.

Theorem. *The group $C_p \times C_{p^k}$ is separable for $p \leq 3$ and $k \geq 1$.*

Actually, we prove that each of the above groups is a Schur group. Recall that a finite group is called a *Schur* group if every Cayley scheme over this group is schurian. This definition was suggested by

Pöschel in [2]. Together with the previously obtained results, this completes the classification of abelian Schur groups of odd order.

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The Weisfeiler-Leman dimension of graphs and isomorphism testing

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The Weisfeiler-Leman algorithm is an indispensable subroutine that appears both in practical and theoretical algorithms for the graph isomorphism problem. More accurately, it comprises a family of algorithms: for every positive integer k there is a k-dimensional version. The 1-dimensional version is often called color refinement or naive vertex refinement and the 2-dimensional version is the classical Weisfeiler-Leman algorithm tightly related to coherent configurations. The k-dimensional variant repeatedly colors k tuples of vertices, collecting more and more information, until after at most n^k iterations no new information is gathered and stabilization occurs. The collected information can often be used to distinguish non-isomorphic graphs.

The Weisfeiler-Leman dimension of a graph is the minimum k for which the k-dimensional Weisfeiler-Leman algorithm distinguishes the graph from every non-isomorphic graph. Trivially this dimension is in $O(n)$ for graphs on n vertices and the famous result of Cai, Fürer and Immerman [1] shows that there are graphs with a Weisfeiler-Leman dimension in $\Omega(n)$.

My talk will touch on several recent results regarding the Weisfeiler-Leman algorithm that are related to isomorphism testing. Specifically, I will address the following:

- The properties of graphs or, more generally, structures that have Weisfeiler-Leman dimension 1. In other words, graphs or structures that can always be distinguished from other non-isomorphic objects using color refinement. While for dimension 2 or higher the analogous question appears not to reveal a structured answer, for dimension 1, i.e., color refinement, a precise answer can be given [2].
- I will also talk about the maximal number of iterations that the 2-dimensional version can take until it reaches stabilization. While a linear lower bound was shown by Fürer [2], the best upper bound of $O(n^2/\log(n))$ shows that the trivial upper bound of $O(n^2)$ is not tight [4].
- Finally, I will discuss the Weisfeiler-Leman dimension of planar graphs, which is at most 3, and explain how this gives simple isomorphism tests for planar graphs [5].

This talk is based on results obtained jointly with Sandra Kiefer, Ilia Ponomarenko, and Erkal Selman.

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Finding a maximal subgroup of minimal index in polynomial time

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Let G be a finite group. Denote by $\kappa(G)$ the smallest positive integer d, such that there exists a nontrivial homomorphism from G into the symmetric group S_d. Clearly $\kappa(G)$ is the smallest index of a maximal subgroup of G. We obtain the following result.

Theorem 1. *Given a Cayley table of a finite group G, the number $\kappa(G)$ can be found in polynomial time in $|G|$.*

In [1], Dutta and Kurur introduced the *group representability problem*: given a group G and a graph X, decide whether there is a nontrivial homomorphism from G into the automorphism group $Aut(X)$ of X. By [Theorem 8, 1] the problem of group representability on trees is polynomial-time Turing reducible to the problem of testing, given an integer d and a group G via Cayley table, whether there is a nontrivial homomorphism to S_d or not. Combining that with our result, we get the following corollary.

Corollary. *The problem of group representability on trees can be solved in polynomial time.*

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Discrete version of Fuglede's conjecture and Pompeiu problem

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We present a discrete version of two old problems (Fuglede and Pompeiu) originated in analysis that were investigated by many researcher and we develop a strong connection between them. This connection help us to provide new results for Fuglede's conjecture in the discrete setting. Pompeiu raised the following question [5]. Take a continuous function f on the plane whose integral on every unit disc is zero. Does it follow that f is the constant zero function? The answer is no, but the question initiated several different type of investigations including many positive results as well. Fuglede conjectured [1] that a bounded domain $S \subset \mathbb{R}^d$ tiles the d-dimensional Euclidean space if and only if the set of $L^2(S)$ functions admits an orthogonal basis of exponential functions. The discrete version of Fuglede's conjecture might be formulated in the following way. A subset S of a finite abelian group G tiles G if and only if the character table of G has a submatrix, whose rows are indexed by the elements of S, which is a complex Hadamard matrix. The spectral-tile direction of Fuglede's conjecture was disproved by Tao [6] and the proof is based on a counterexample for elementary abelian p-groups of finite rank. This result led to the first counterexample for the original problem in the continuous case. The other direction was disproved by Koloutzakis and Matolcsi [3].

In order to find answers for Pompeiu type problems one has to investigate the eigenvalues of the Cayley graph $Cay(G, S)$, that shows the connection of these two problems. It is worth to investigate Fuglede's conjecture for finite cyclic groups since every tiling of $\mathbb{Z}$ is periodic so it originates in a tiling of a finite cyclic group. However, not much is known for cyclic groups. A recent paper of Malikiosis and Kolountzakis [4] shows that Fuglede's conjecture holds for cyclic group of order $p^n q$, where p and q are different primes. Our main contribution towards Fuglede's conjecture for cyclic groups is to connect this problem with the Pompeiu problem, introduce more combinatorial ideas and verify it for yet unknown cases: cyclic groups of order p^2q^2 and pqr, where r is also a prime. Further we give a neat and combinatorial proof for the previously known fact (proved by Iosevich, Mayeli and Pakianathan [2]) that Fuglede's conjecture holds for $\mathbb{Z}_p^2$.

A joint work with Gergely Kiss and Máté Vizer.

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Constructing Majorana representations

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Majorana theory is an axiomatic framework in which to study objects related to the Monster group and its $196,884$ dimensional representation, the Griess algebra. The objects at the centre of the theory are known as Majorana algebras and can be studied either in their own right, or as Majorana representations of certain groups. I will discuss my work developing an algorithm in GAP to construct the Majorana representations of a given group. This work is based on a paper of Á. Seress and is joint with M. Pfeiffer.

Short presentations

2-closure of $\frac{3}{2}$-transitive groups in polynomial time

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Let G be a permutation group on a finite set Ω. Wielandt [6] defined the 2-closure $G^{(2)}$ of G to be the automorphism group of the set of 2-orbits of G. There is a natural Galois correspondence between permutation groups and coherent configurations (see, e.g. [2]). In the context of computational complexity theory, this correspondence leads to two natural problems: given a coherent configuration X, find the schurian closure of $\mathrm{Aut}(X)$ and given a permutation group G, find $G^{(2)}$. It is well known that the first problem is equivalent to the Graph Isomorphism Problem. Here we are interested in the second one, which we specify as follows.

2-Closure problem. Given a finite permutation group G, find generators of its 2-closure $G^{(2)}$.

Clearly this problem is tightly connected with the problem of finding the automorphism group of a schurian coherent configuration. It was solved for nilpotent groups [5] and groups of odd order [3] in time polynomial in $|\Omega|$ by using a technique from [1] and the fact that the 2-closures of such groups are solvable.

In the talk (based on a joint paper with Andrey Vasil′ev), we discuss this problem for $\frac{3}{2}$-transitive groups. It turns out that the recently finished classification of finite $\frac{3}{2}$-transitive groups [4] provides sufficient theoretical tools for proving the following

Theorem. *The 2-closure problem for a $\frac{3}{2}$-transitive group of finite degree n can be solved in time polynomial in n.*

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Cubic bi-Cayley graphs over solvable groups are 3-edge-colorable

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Bi-Cayley graphs are graphs admitting a semiregular group of automorphisms with two orbits. A notable cubic subclass of bi-Cayley graphs is the so-called generalized Petersen graphs. Castagna and Prins proved in 1972 that all generalized Petersen graphs except for the Petersen graph itself can be properly 3-edge-colored. In this talk, we are going to discuss the extension of this result to all connected cubic bi-Cayley graphs over solvable groups. Our theorem is a bi-Cayley analogue of similar results obtained by Nedela and Škoviera for Cayley graphs any by Potočnik for vertex-transitive graphs.

Average mixing of quantum walks on graphs

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The study of quantum walks on graphs has given rise to a rich connection between algebraic graph theory, linear algebra and quantum computing. A system of interacting quantum qubits can be modelled by a graph. The evolution of the quantum system can be completely encoded as a quantum walk in a graph, which can be seen, in some sense, as a quantum analogue of random walk. The behaviours of the quantum walks are seen as graph invariants and some properties have even been proposed as complete graph invariants.

In this talk, I will present recent results on the average mixing matrix of a graph: a quantum walk has a transition matrix which is a unitary matrix with complex values and thus will not converge, but we may speak of an average distribution over time, which is modelled by the average mixing matrix. It is a surprising fact that this matrix is equal to the sum of the Schur squares of the idempotent projections in the spectral decomposition of the adjacency matrix. We take advantage of this to approach several problems using algebraic graph theory.

Automorphisms of an $\mathrm{srg}(162, 21, 0, 3)$

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Maknev and Nosov [1] proved that if g is an automorphism of a strongly regular graph with parameters $(162, 21, 0, 3)$ then the subgraph fixed by g is either empty or $K_{1,3}$. And if there are no fixed vertices then an srg(81,20,1,6) can be obtained by collapsing the orbits. We present a computation showing that an automorphism of order 2 can not be without fixed vertices, and we show that an $\mathrm{srg}(162, 21, 0, 3)$ is not vertex-transitive.

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Resistance-distance transform in the context of Weisfeiler-Leman stabilization

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The concept of *resistance distance* (RD), which was proposed in [1], originates from mathematical chemistry. Inspired by the effective resistance of electrical circuits, it found interesting applications in diverse fields, including AGT. More recent works [2, 3] proposed a somewhat shortcut method for obtaining RD between any pair of vertices for a wide class of graphs. Based on these ideas, we suggest a procedure, called *resistance-distance transform* (RDT), which can be applied to any simple graph Γ, as well as to a complete undirected graph with colored edges. When acting on graph Γ, RDT attributes to each edge (and non-edge) its corresponding RD value. Thus the outcome of this procedure constitutes a complete symmetric colored graph and can be compared to the result of Weisfeiler-Leman stabilization (WLS). Surprisingly, for many classes of experimentally inverstigated graphs, just one iteration of RDT provides the same result as WLS. We shall discuss the observed links between the two procedures obtained via both theoretical and computer aided considerations. In particular, the classical WL closure of graph Γ, expressed in the matrix form [4], will be compared with the RD matrix expressed either using the Moore-Penrose pseudo-inverse of the Laplacian matrix of Γ [1] or the shortcut methods of [2, 3]. This is a joint project together with Mikhail Klin (Ben-Gurion University, Israel).

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Using strong paths to solve isomorphism problems

Matthias Koch

In this presentation a universal strategy to solve isomorphism problems is developed. The strategy is based on the homomorphism principle for group actions [1], double cosets and the “Snakes and Ladders” algorithm [4].

Many discrete structures can be built from simpler ones by using the fundamental homomorphism theorem applied to group actions [3]. For a wide range of discrete structures this is not applicable, since there is no suitable unidirectional chain of homomorphisms. Following the ’snakes and ladders’ strategy by using homomorphisms in both directions, an appropriate chain of homomorphisms (ladder) can be built [5].

The original snakes and ladders algorithm had several drawbacks, namely the extensive demand on memory and the inadequate runtime when used for single structures. These downsides have now been overcome by extending the mathematical basis by the so-called “Strong Paths” [2].

This strategy is applicable for the construction of double cosets. Since many isomorphism problems can be expressed by means of double cosets they can be solved by this approach [5].

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Invariants for efficiently computing the autotopism group of a partial latin rectangle

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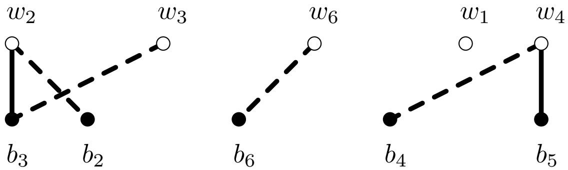
A joint work with Raúl M. Falcón and Rebecca J. Stones.

Let $[n] := \{1, \ldots, n\}$. An $r \times s$ *partial Latin rectangle based on* $[n]$ is an $r \times s$ array $L = (L[i,j])$ containing symbols from the set $[n] \cup \{\cdot\}$ such that each row and each column contains at most one copy of any symbol in $[n]$. The set of such rectangles is denoted $\mathrm{PLR}(r, s, n)$. An *isotopism* $\theta = (\alpha, \beta, \gamma) \in S_r \times S_s \times S_n$ acts on the set $\mathrm{PLR}(r, s, n)$, by permuting the rows, columns, and symbols of any $L \in \mathrm{PLR}(r, s, n)$ by α, β, and γ, respectively. If $\theta(L) = L$, then θ is said to be an *autotopism* of L. The set $\mathrm{Atop}(L)$ of autotopisms of L forms a group, called the *autotopism group* of L. For a Latin square L, the autotopism group $\mathrm{Atop}(L)$ has received much attention, such as early works of Schönhardt [10] and Artzy [1] and, more recently, Bailey [4]. Thereafter, many works of McKay, Wanless and others deal with the computation of $\mathrm{Atop}(L)$. Autotopisms of (partial) Latin rectangles, however, have only been studied recently by Falcon and Stones.

Any method for obtaining the autotopism group of a (partial) Latin square or rectangle is based on a backtracking computation of non-polynomial time complexity, or on the computation of the automorphism group of a related graph. McKay et al. [9] introduced the standard way of computing the autotopism group $\mathrm{Atop}(L)$ of a Latin square L of order n by constructing a vertex-coloured graph whose automorphism group is isomorphic to $\mathrm{Atop}(L)$. The problem of computing the autotopism group of a Latin square is, therefore, as difficult as solving the graph isomorphism problem, whose complexity is $\exp(O(\sqrt{n \log n}))$ by Babai, Kantor and Luks [3] and even claimed to be $\exp((\log n)^{O(1)})$ by Babai [2], where n is the number of vertices. This method has recently been generalized in a natural way [5] so that any $r \times s$ partial Latin rectangle with n symbols and m filled cells is uniquely related to a vertex-coloured bipartite graph with $m+r+s+n$ vertices and $3m$ edges.

In order to narrow down the search by backtracking as much as possible or reduce the complexity of computing the automorphism group of the previously described graphs as low as possible, distinct autotopism invariants of (partial) Latin rectangles have been described [5, 6, 7, 8, 11] for which the complexity of their computation is polynomial in the order of the array under consideration. All these invariants yield a series of partitions of the entries, rows, columns and/or symbols of the corresponding (partial) Latin rectangle so that all their parts are preserved by autotopisms. The finer each partition is, the lower the complexity of computing the autotopism group is. Nevertheless, no autotopism invariant is currently known so that the corresponding partitions are optimal.

In this work we introduce new autotopism invariants which get closer to optimal partitions. For this we define the (r_1, r_2)-*row graph* of an $r \times s$ partial Latin rectangle L, with $r \geq 2$, as a vertex-and-edge-colored bipartite graph related to its r_1^{th} and r_2^{th} rows. For example, the graph corresponding to the rows $[2, \cdot, 3, \cdot, 6, \cdot, 1, 4]$ and $[3, 2, \cdot, 6, \cdot, 4, \cdot, 5]$ is



The distribution of these graphs into isomorphism classes yields three new autotopism invariants that can be either combined with other invariants or used in those cases for which the currently known invariants have little or no effect for computing the autotopism group of the partial Latin rectangle under consideration. Experimental evidence shows that these three invariants yield partitions that are closer to optimal, thus improving the computational complexity of finding autotopism groups of random partial Latin rectangles.

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Inverse problems in spectral theory of graphs

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Spectral graph theory uses the eigenvalues of matrices associated with a graph to determine structural properties of that graph. Graphs with the same spectrum are called cospectral. The construction of the cospectral graphs calling GM-switching was discussed in the paper E. R. van Dam and W. H. Haemers "Which graphs are determined by their spectrum?" (2003). We consider this construction for graph G taking the cycle C_{2n} and adjoining a vertex v adjacent to half the vertices of C_{2n}. For these graphs for small n are determined the pairs of cospectral nonisomorphic graphs.

On the complexity of testing isomorphism of graphs of bounded eigenvalue multiplicity

Takunari Miyazaki
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In this talk, using Delsarte, Goethals and Seidel's fundamental theorem of spherical codes and designs, I will make some observations about the complexity of graph-isomorphism testing. In particular, I will derive a set of group-theoretic conditions under which testing isomorphism of graphs of bounded eigenvalue multiplicity is immediately reducible to testing isomorphism of graphs of bounded color multiplicity.

Steinitz's lattice and diagonal constructions

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Steinitz's lattice was introduced at the beginning of the XX century by A. Steinitz for describing the structure of subfields of algebraically closed field of prime characteristic. It can be determined as the lattice of supernatural numbers with a relation of the divisibility.

In this talk we discuss Steinitz's lattices arising from diagonal constructions of groups, semigroups, Boolean algebras and metric spaces and consider connections between them.

Combinatorics in sublattices of invariant subspaces

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Joint work with David Mingueza and M. Eulàlia Montoro[2]

Let $\mathbb{F}$ be a field. Given a matrix $A \in M_n(\mathbb{F})$, a subspace $V \subset \mathbb{F}^n$ is A-invariant if $AV \subset V$. An A-invariant subspace is called characteristic (respectively hyperinvariant) if it is also T-invariant for all of the nonsingular matrices T (respectively, matrices T) commuting with A. We denote by $Inv(A)$, $Chinv(A)$ and $Hinv(A)$ the lattices of invariant, characteristic and hyperinvariant subspaces, respectively. Obviously, $Hinv(A) \subset Chinv(A) \subset Inv(A)$.

The lattice $Inv(A)$ is not necessarily finite, but $Chinv(A)$ is always finite. We present here how to obtain the cardinality of $Chinv(A)$.

When the minimal polynomial of A is separable, the study of the lattices $Inv(A)$, $Hinv(A)$ and $Chinv(A)$ can be reduced to the case where A is nilpotent (see [2, 3, 5] respectively). It was proved in [**?**] that only if $\mathbb{F} = GF(2)$, the lattices of characteristic and hyperinvariant subspaces may not coincide.

Assume then that A is a nilpotent matrix, $\mathbb{F} = GF(2)$ and that the minimal polynomial of the endomorphism splits over the underlying field $\mathbb{F}$. Shoda's Theorem [6] characterizes the existence of characteristic non hyperinvariant subspaces. According to [4], a subspace $X \in Chinv(A) \setminus Hinv(A)$ can be written as a direct sum of two subspaces $X = Y \oplus Z$, with Y and Z associated to a so called chartuple, Y is hyperinvariant with some extra conditions and Z is called a minext subspace.

The cardinality of $Hinv(A)$ is known. We understand

$$Chinv(A) = Hinv(A) \cup (Chinv(A) \setminus Hinv(A)).$$

To compute the cardinality of $Chinv(A) \setminus Hinv(A)$ we find the number of possible chartuples, and the number of minext and hyperinvariant subspaces associated to each chartuple. We present here the highly combinatorial results. The cardinality of minext subspaces is stated in terms of a recurrent formula, and the cardinality of hyperinvariant subspaces associated to a chartuple, through an algorithm.

The results obtained involve combinatorial numbers and Gauss binomial coefficients. The algorithm constructs a table which generalizes the Pascal matrix. It is shown that the results of the algorithm can also be derived from generating polynomials. A different combinatorial strategy to obtain the result will also be presented.

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The construction of combinatorial structures and linear codes from orbit matrices of strongly regular graphs

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Orbit matrices of strongly regular graphs were introduced in 2011 by M. Behbahani and C. Lam [1].

A method for constructing self-orthogonal codes from orbit matrices of strongly regular graphs admitting an automorphism group G which acts with orbits of length w, where w divides $|G|$ is given in [2]. In this talk we will present the construction of some combinatorial structures and linear codes from orbit matrices of strongly regular graphs and their submatrices.

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Coherent configurations and higher dimensional Weisfeiler-Leman equivalence

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The k-Weisfeiler-Leman refinements for a family of algorithms indexed by a positive integer k which approximate graph isomorphism. For $k = 2$, the algorithm can be formulated in terms of an isomorphism problem of *coherent algebras*. We make this connection explicit for larger values of k, and show that for $k = 2m$, $m > 1$ as well, the algorithm can be understood in terms of coherent algebra isomorphism. This allows us to make a connection with a known graph invariant arising from quantum in formation: indeed, we show that the k-*boson* invariant defined in [1] fails to distinguish Cai-Fürer-Immerman graphs. Finally, we give a tight connection between k-*invertible map* refinements [1] and k-Weisfeiler-Leman.

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Reconstructability index of finite 2-step nilpotent graph p-groups

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Let $\Gamma = (T, E)$ be a graph. Let V be the vector space freely generated by the set of vertices $T = \{v_1, \ldots, v_n\}$ over finite field $\mathbb{F}_p$ with $p \neq 2$. In [3] we define the free 2-step nilpotent Lie algebra $N_n = V \bigoplus \wedge^2 V$ and define $M = \mathrm{G}(N_n^R)$ to be the group Lazard corresponding to Lie ring N_n^R.

BCH-formula. Because any Lie ring homomorphism $\varphi : L_1^R \to L_2^R$, where $L_1^R, L_2^R \in \mathcal{L}$, induces the group homomorphism $\hat{\varphi} : \mathrm{G}(L_1^R) \to \mathrm{G}(L_2^R)$. Let M_n is a free group freely generated by T in the variety of groups determined by the identities $x^p = 1, [[x, y], z] = 1$. Then a 2-*step nilpotent finite graph p-group* for the graph $\Gamma = (T, E)$ is defined as

$$G_\Gamma = M_n/J,$$

where J is a normal subgroup of M_n generated by $v_i \wedge v_j$ for all $\{v_i, v_j\} \in E$. 2-step nilpotent). It is shown in [3] that isomorphism of graphs Γ_1 and Γ_2 implies isomorphism of their graph groups and vice versa.

Following [7], let *multiset* S on a ground set X be a function $m : X \to \mathbb{N}$, and let $\mathcal{P}(S)$ and $\mathcal{P}^{\{k\}}(X)$ denote collections of all multisets contained in S of any size and of size k respectively. Let $G \to X$ be a group action on X. Then G acts naturally on multisets S on X by $(g, S) = \{(g, x) : x \in S\}$. Two multisets S, T are called *isomorphic w.r.t.* G if there exists $g \in G$ such that $(g, S) = T$; the isomorphism class of S under given group action is denoted by $[S]_G$. Then the k*-deck* of S is defined as the multiset

$$D_k(S) = \{[K]_G \mid K \in \mathcal{P}(S), |K| \leq k\},$$

containing isomorphism classes of r-subsets of S with $r \leq k$. A multiset S in X is called k*-reconstructable* if every multiset $T \subset X$ with $D_k(T) = D_k(S)$ is isomorphic to S.

Reconstructability index $r_{\mathbb{N}}(G \to X)$ *of a group action* $G \to X$ is the minimum k such that all finite multisets in X are k-reconstructable; $r_{\mathbb{N}}(G)$ denotes the reconstructability index of a left-regular action $G \to G$.

In this work of we show that $r_{\mathbb{N}}(G_\Gamma) \leq 36$ for any 2-step nilpotent finite graph p-group G_Γ.

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Symmetry vs. Regularity
Book of abstracts

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