Graph switching, 2-ranks, and graphical Hadamard matrices

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Joint work with S. Butler and W.H. Haemers

Symmetry vs Regularity
Pilsen, 1-7 July 2018

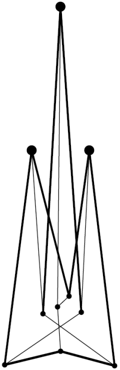
Plan

Background

Part I

Part II

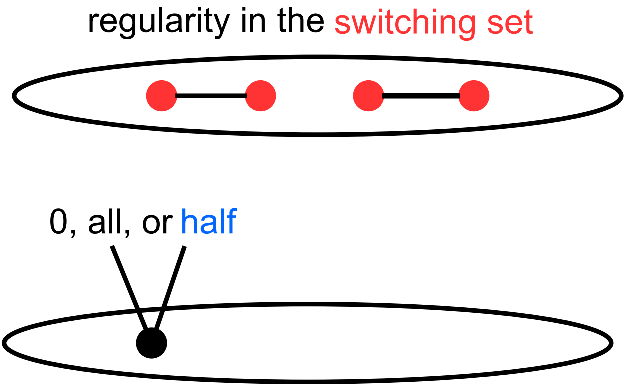
Graph, adjacency matrix and spectrum



$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

spectrum: $3, 1, 1, 1, 1, 1, -2, -2, -2, -2$

Godsil-McKay switching



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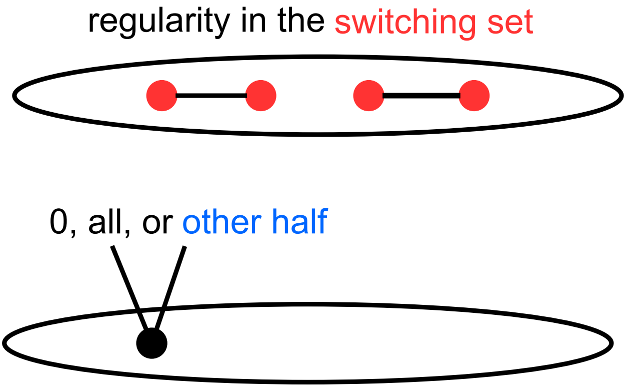

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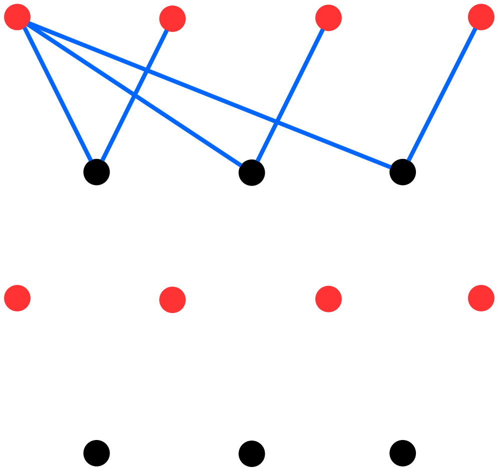

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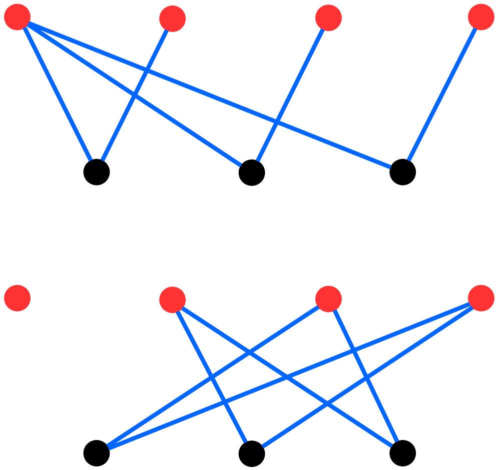

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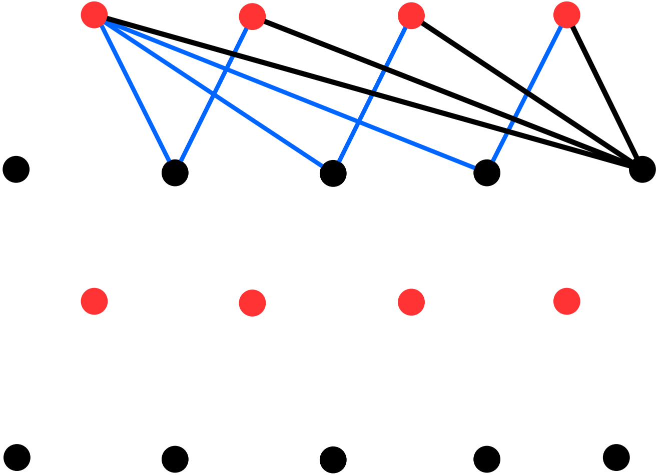

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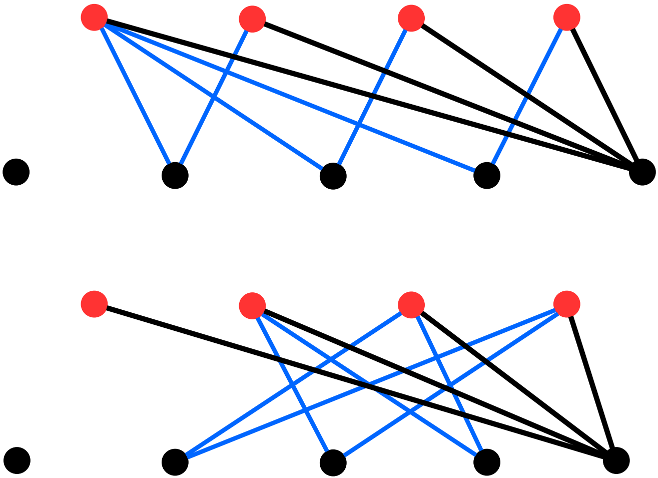

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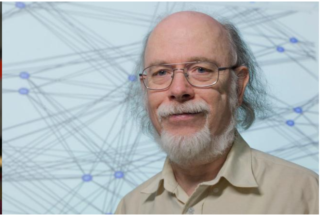

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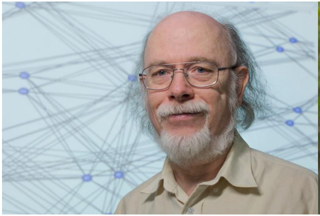 

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Part II

Main question:

Can we construct new SRGs with the same parameters as the symplectic graph?

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Joint work with W.H. Haemers

2-rank

The 2-rank of a graph is the rank of its adjacency matrix over the finite field $\mathbb{F}_2$.

Godsil-McKay switching and its 2-rank behaviour

$$A = \begin{array}{|c|ccc|}\hline B & \multicolumn{1}{c|}{N} & \multicolumn{1}{c|}{J} & O \\ \hline N^\top & & & \\ \cline{1-1} J & & C & \\ \cline{1-1} O & & & \\ \hline\end{array} \qquad A' = \begin{array}{|c|ccc|}\hline B & \multicolumn{1}{c|}{\bar{N}} & \multicolumn{1}{c|}{J} & O \\ \hline \bar{N}^\top & & & \\ \cline{1-1} J & & C & \\ \cline{1-1} O & & & \\ \hline\end{array}$$

$$S = \begin{array}{|c|ccc|}\hline O & \multicolumn{1}{c|}{J} & \multicolumn{1}{c|}{O} & O \\ \hline J & & & \\ \cline{1-1} O & & O & \\ \cline{1-1} O & & & \\ \hline\end{array}$$

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$$S = \left[\begin{array}{c|ccc} O & J & O & O \\ \hline J & & & \\ O & & O & \\ O & & & \end{array}\right]$$

$$A' = A + S \pmod 2$$

Godsil-McKay switching and its 2-rank behaviour

Lemma (Haemers, Peeters and van Rijckevorsel 1999)

The 2-rank of a symmetric integral matrix with zero diagonal is even.

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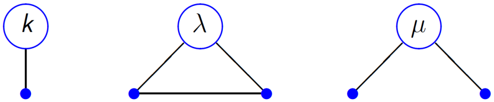
Lemma (Abiad and Haemers 2016)

Suppose 2-rank$(A) = r$*, then* r *is even and 2-rank*$(A') = r - 2$*,* r *or* $r + 2$*.*

G: SRG



G': graph obtained from G by switching

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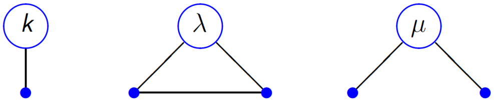

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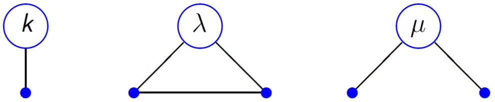

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$\Downarrow$

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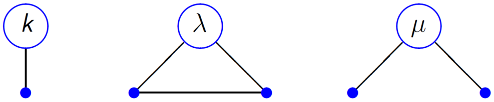

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G, G' same parameters (n, k, λ, μ)

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OUR TOOL: 2-rank

The symplectic graph

The symplectic graph $Sp(2\nu, 2)$ is a SRG with parameters

$$P_0(\nu) = \left(2^{2\nu} - 1,\ 2^{2\nu-1},\ 2^{2\nu-2},\ 2^{2\nu-2}\right).$$

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Theorem (Peeters 1995)

$P_0(\nu) = Sp(2\nu, 2)$ is characterized by its parameters and the minimality of its 2-rank, which equals 2ν.

Switched symplectic graphs

Godsil-McKay switching set

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ z \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ z \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ z \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ z \end{bmatrix}$$

where $z \in \mathbb{F}_2^{2\nu-6}$.

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where $z \in \mathbb{F}_2^{2\nu-6}$.

Lemma (Abiad and Haemers 2016)

The set $B = \{v_1, v_2, v_3, v_4\}$ is a Godsil-McKay switching set of $Sp(2\nu, 2)$ for $\nu \geq 3$.

Switched symplectic graphs

Theorem (Abiad and Haemers 2016)

For $\nu \geq 3$, the graph G' obtained from $Sp(2\nu, 2)$ by switching with respect to the switching set B given above, is strongly regular with the same parameters as $Sp(2\nu, 2)$, but with 2-rank equal to $2\nu + 2$.

Repeated switching in $Sp(6, 2)$

We ran a search of repeated switching in $Sp(6, 2)$ and found ≥ 1800 nonisomorphic SRG $(63, 32, 16, 16)$ with 2-ranks:

$$6, 8, \ldots, 18.$$

Hadamard matrices: $HH^\top = nI$

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & - & - \\ 1 & - & 1 & - \\ 1 & - & - & 1 \end{bmatrix}$$

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Hadamard matrices: $HH^\top = nI$

$$H = \begin{bmatrix} \color{red}{1} & 1 & 1 & 1 \\ 1 & \color{red}{1} & - & - \\ 1 & - & \color{red}{1} & - \\ 1 & - & - & \color{red}{1} \end{bmatrix}$$

graphical

Hadamard matrices: $HH^\top = nI$

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & - & - \\ 1 & - & 1 & - \\ 1 & - & - & 1 \end{bmatrix} \qquad A_H = \frac{1}{2}(J - H)$$

If H normalized, A_H corresponds to $\left(n-1, \frac{n}{2}, \frac{n}{4}, \frac{n}{4}\right)$:

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Hadamard matrices and 2-ranks

H_1, H_2 Hadamard matrices $\Longrightarrow H_1 \otimes H_2$ Hadamard matrix.

Lemma (Abiad and Haemers 2016)

Let H_1 and H_2 be Hadamard matrices, and let $\rho(H) = \text{2-rank}(A_H)$. Then,

$$\rho(H_1 \otimes H_2) \leq \rho(H_1) + \rho(H_2),$$

with equality if H_1 and H_2 are normalized.

Alternative description of $Sp(2\nu, 2)$ using a recursive construction

Take

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & - & - \\ 1 & - & 1 & - \\ 1 & - & - & 1 \end{bmatrix}, \quad \text{then} \quad A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \Longleftarrow Sp(2,2)$$

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$$H^{\otimes \nu} = H \otimes H \otimes \cdots \otimes H \quad (\nu \text{ times})$$

which is a normalized graphical Hadamard matrix of order 4^ν and 2-rank$(A_{H^{\otimes \nu}}) = 2\nu$.

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Theorem [Peeters, 1995]
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Theorem [Peeters, 1995]
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The SRG associated with $H^{\otimes\nu}$ is $Sp(2\nu, 2)$

Hadamard matrices and 2-ranks

In $H^{\otimes\nu}$, we can replace any $H \otimes H \otimes H$ by any other regular graphical Hadamard matrix of order 64 coming from the SRG of order 63 found by computer (with 2-ranks 6,8,10,12,14,16,18).

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Corollary (Abiad and Haemers 2016)

Using the above recursive construction we get SRG with the parameters of $Sp(2\nu, 2)$ and 2-ranks:

$$2\nu,\ 2\nu+2, \ldots, 2\nu+12\lfloor \nu/3 \rfloor.$$

Related results

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  Switched graphs from quadrics in $PG(n, 2)$

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- Munemasa and Vanhove (2018+)
  Construction of graphs with the same parameters as $Sp(2\nu, 2)$ and 2-rank at least 4ν

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Can we construct SRGs (from graphical Hadamard matrices) having the same parameters but different 2-rank?

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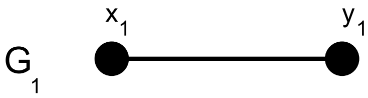
2-rank=26???

A graph product $\otimes$



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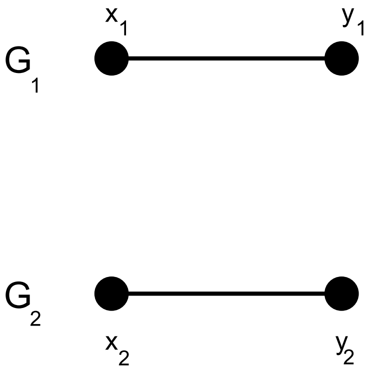

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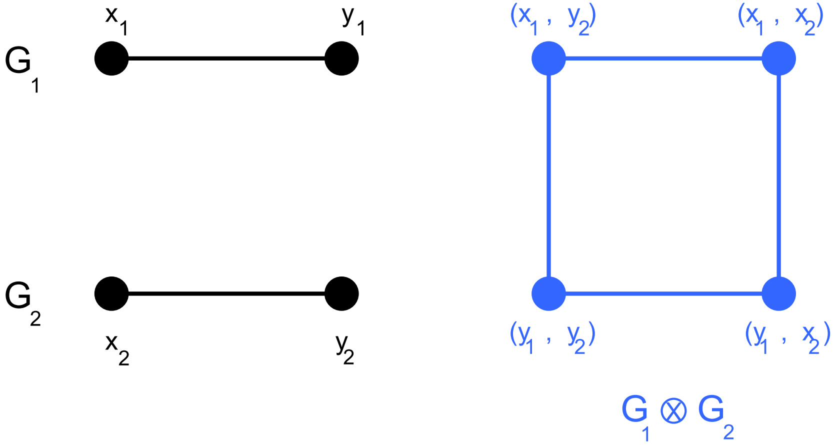

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$G_1 \otimes G_2$ has vertex set $V_1 \times V_2$, where two vertices (x_1, x_2) and (y_1, y_2) are adjacent **whenever**:

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$\{x_i, y_i\}$ are adjacent in G_i for $i = 1, 2$,
or
$\{x_i, y_i\}$ are nonadjacent in G_i for $i = 1, 2$.

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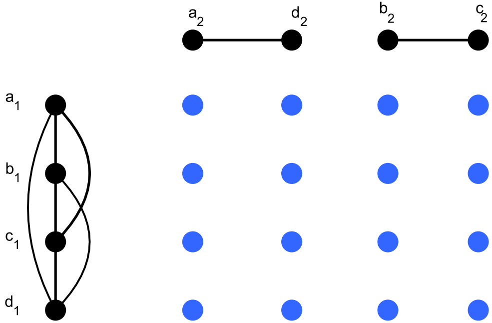

Clebsch graph: $2K_2 \otimes K_4 = (16,10,6,6)$

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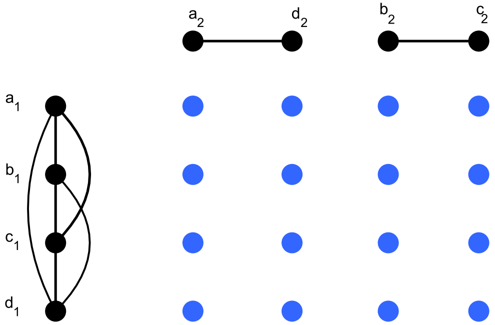

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$$H_{2K_2} = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix} \qquad H_{2K_2 \otimes K_4} = \begin{bmatrix} H & H & H & -H \\ H & H & -H & H \\ H & -H & H & H \\ -H & H & H & H \end{bmatrix}$$

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Inspired by the Kronecker product of Hadamard matrices...

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If H_1 and H_2 are graphical Hadamard matrices, then

$$G_{H_1} \otimes G_{H_2} = G_{H_1 \otimes H_2}$$

Our tool: graph product and its 2-rank behaviour

Theorem (Abiad, Butler and Haemers 2018)

For two graphs G_1 and G_2 the following hold:

(i) $\mathbf{1} \in \mathrm{Col}_2(G_1 \otimes G_2)$ *if and only if* $\mathbf{1} \in \mathrm{Col}_2(G_1)$ *or* $\mathbf{1} \in \mathrm{Col}_2(G_2)$,

(ii) *if* $\mathbf{1} \in \mathrm{Col}_2(G_1)$ *and* $\mathbf{1} \in \mathrm{Col}_2(G_2)$ *then*

$$\text{2-rank}(G_1 \otimes G_2) = \text{2-rank}(G_1) + \text{2-rank}(G_2) - 2,$$

(iii) *if* $\mathbf{1} \notin \mathrm{Col}_2(G_1)$ *or* $\mathbf{1} \notin \mathrm{Col}_2(G_2)$ *then*

$$\text{2-rank}(G_1 \otimes G_2) = \text{2-rank}(G_1) + \text{2-rank}(G_2).$$

SRGs

Graph	(n, k, λ, μ)	2-rank
K_4	(4,3,2,0)	4
$2K_2$	(4,1,0,0)	4
Lattice graph $L(4) = 2K_2 \otimes 2K_2$	(16,6,2,2)	6
Shrikhande graph=switched $L(4)$	(16,6,2,2)	6
Clebsch graph$=2K_2 \otimes K_4$	(16,10,6,6)	6

switched SRGs: $P_0(3)$, $P_\pm(3)$

Graph	$(\boldsymbol{n}, \boldsymbol{k}, \boldsymbol{\lambda}, \boldsymbol{\mu})$	**2-ranks**
Sp(6,2)	(63,32,16,16)	$\{6, 8, \ldots, 24\}$
Clebsch graph $\otimes\, 2K_2 = 2K_2 \otimes K_4 \otimes 2K_2$	(64,36,20,20)	$\{8, 10, \ldots, 26\}$
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- Seidel switching on $Sp(6,2)$ gives SRGs with the same parameters but it does not change the 2-rank!
- $Sp(6,2)$ has GM switching sets that increase the 2-rank after GM switching

switched SRGs: $P_0(3)$, $P_\pm(3)$

Graph	$(\boldsymbol{n}, \boldsymbol{k}, \boldsymbol{\lambda}, \boldsymbol{\mu})$	**2-ranks**
Sp(6,2)	(63,32,16,16)	$\{6, 8, \ldots, 24\}$
Clebsch graph $\otimes\, 2K_2 = 2K_2 \otimes K_4 \otimes 2K_2$	(64,36,20,20)	$\{8, 10, \ldots, 26\}$
Shrikhande graph $\otimes\, 2K_2$	(64,28,12,12)	$\{8, 10, \ldots, 26\}$
$L(4) \otimes 2K_2 = 2K_2 \otimes 2K_2 \otimes 2K_2$	(64,28,12,12)	$\{8, 10, \ldots, 26\}$
Shrikhande graph $\otimes\, K_4$ *	(64,36,20,20)	$\{8, 10, \ldots, 26\}$

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$P_+(3) = (64, 36, 20, 20)$
$P_-(3) = (64, 28, 12, 12)$

SRGs with parameters $P_0(\nu)$, $P_\pm(\nu)$

$$P_0(\nu) = Sp(2\nu, 2) = (2^{2\nu} - 1,\ 2^{2\nu-1},\ 2^{2\nu-2},\ 2^{2\nu-2})$$

$$P_\pm(\nu) = (2^{2\nu},\ 2^{2\nu-1} \pm 2^{\nu-1},\ 2^{2\nu-2} \pm 2^{\nu-1},\ 2^{2\nu-2} \pm 2^{\nu-1})$$

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Theorem (Abiad, Butler and Haemers 2018)

(i) *There exist SRGs with parameter set $P_0(\nu)$ and 2-rank r for every even $r \in [2\nu, 2(\nu + 9\lfloor \frac{\nu}{3} \rfloor)]$.*

(ii) *There exist SRGs with parameter set $P_+(\nu)$ and 2-rank r for every even $r \in [2(\nu+1), 2(\nu + 1 + 9\lfloor \frac{\nu}{3} \rfloor)]$.*

(iii) *There exist SRGs with parameter set $P_-(\nu)$ and 2-rank r for every even $r \in [2(\nu+1), 2(\nu + 1 + 9\lfloor \frac{\nu}{3} \rfloor)]$.*

Proof main idea

- Do the switched graphs have the all-one vector in the span of the adjacency matrix?

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- Do the switched graphs have the all-one vector in the span of the adjacency matrix?
- Combine the graph product and the computational search of GM switching sets that increase the 2-rank

Consequence I

Two Hadamard matrices are *equivalent* if one can be obtained from the other by row and column permutation and multiplication of rows and columns by -1.

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Corollary (Abiad, Butler and Haemers 2018)

The number of nonequivalent graphical Hadamard matrices of order 4^{ν} *is unbounded.*

Consequence II

SRGs with parameters $P_{-}(m)$ are known as *maximal energy graphs*.

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Corollary (Abiad, Butler and Haemers 2018)

The number of nonisomorphic maximal energy graphs of order 4^{ν} *is unbounded.*

Open problems

- Does $Sp(6, 2)$ with 2-rank=26 exist?

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- Does $Sp(6, 2)$ with 2-rank=26 exist?
- Can we apply similar techniques to other SRGs?
- Study the 2-rank behaviour of other products used for the construction of Hadamard matrices.

Thank you for listening!