# Automorphisms of an srg(162,21,0,3)

# Distance regular 2-cover of srg(81,10,1,6)

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Pilsen, July 2018

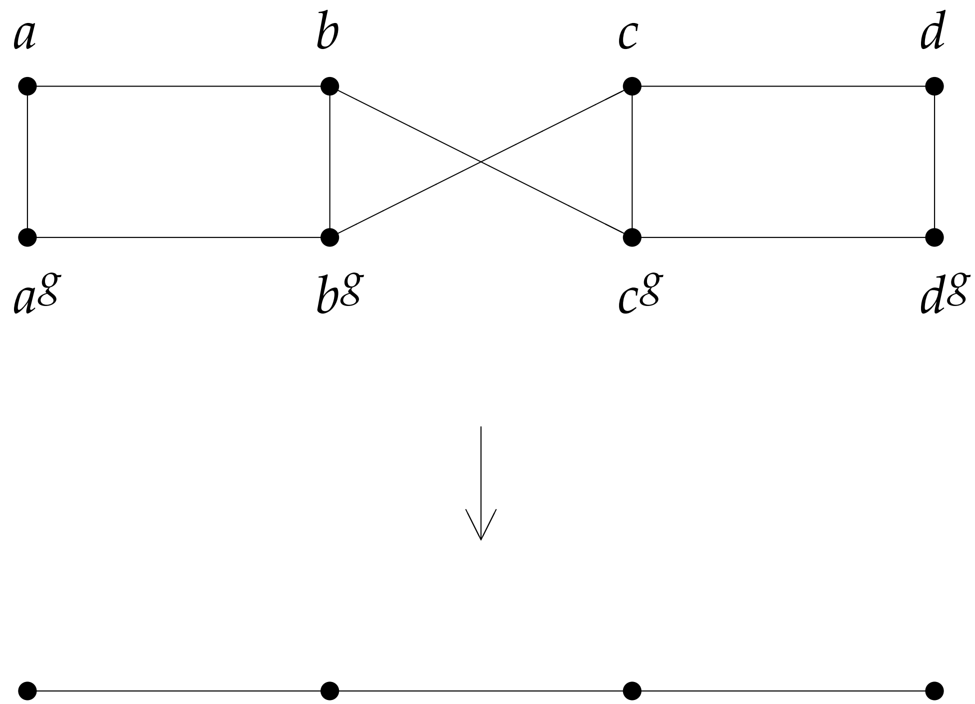
**Theorem**, Makhnev and Nosov 2010
Let $g$ be an automormorphism of srg(162,21,0,3) of order 2.
The either

- Fix$(g) = K_{1,3}$ or
- Fix$(g) = \emptyset$ and vertices $a$ and $a^g$ are adjacent.
  Identifying each pair $\{a, a^g\}$ gives the unique srg(81,20,1,6).

Every automorphism of order 2 is an odd permutation.

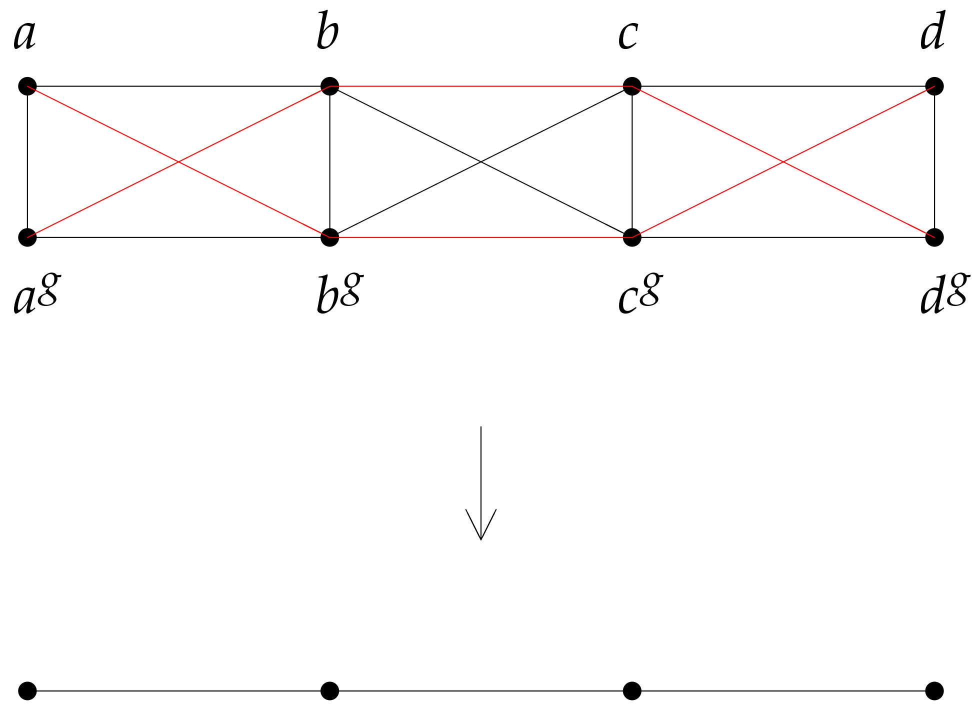
If srg(162,21,0,3) is vertex transitive then every automorphism $g$ of order 2 has Fix$(g) = \emptyset$.

srg(162,21,0,3)




srg(81,20,1,6)

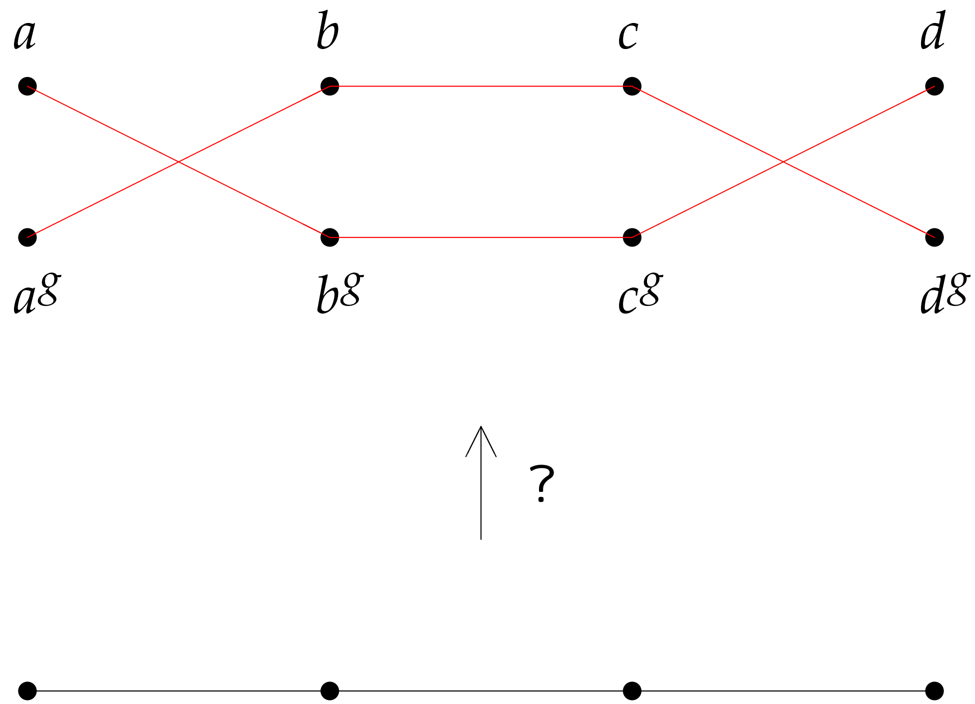
srg(162,21,0,3)




srg(81,20,1,6)

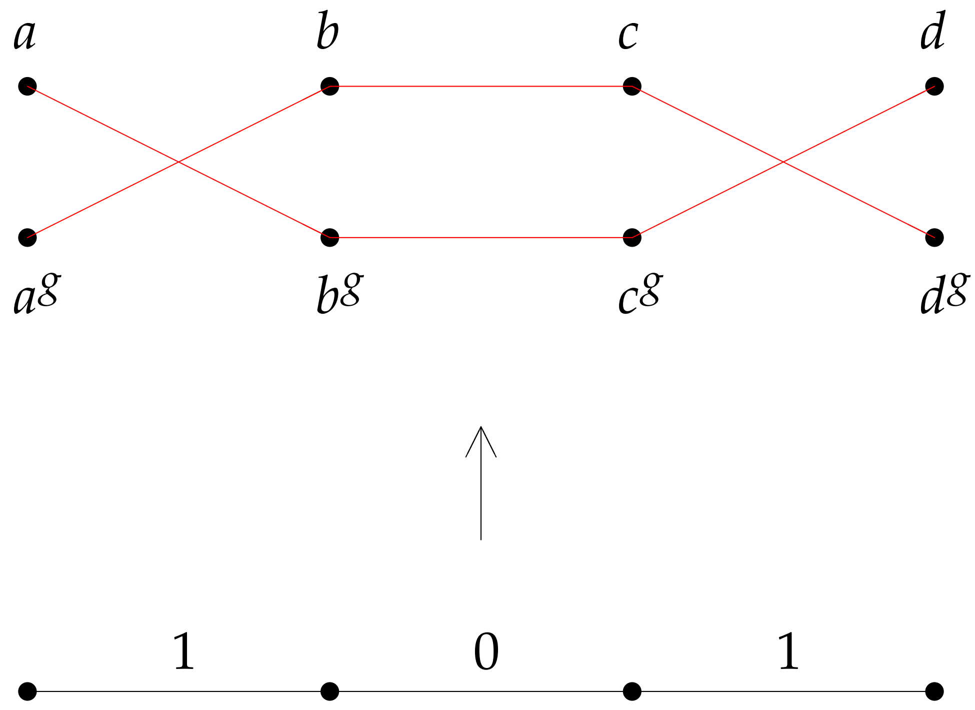
$\{20, 18, 3, 1;\ 1, 3, 18, 20\}$, a 2-cover of srg(81,20,1,6)

2-cover of srg(81,20,1,6)




srg(81,20,1,6) unique Brouwer-Haemers

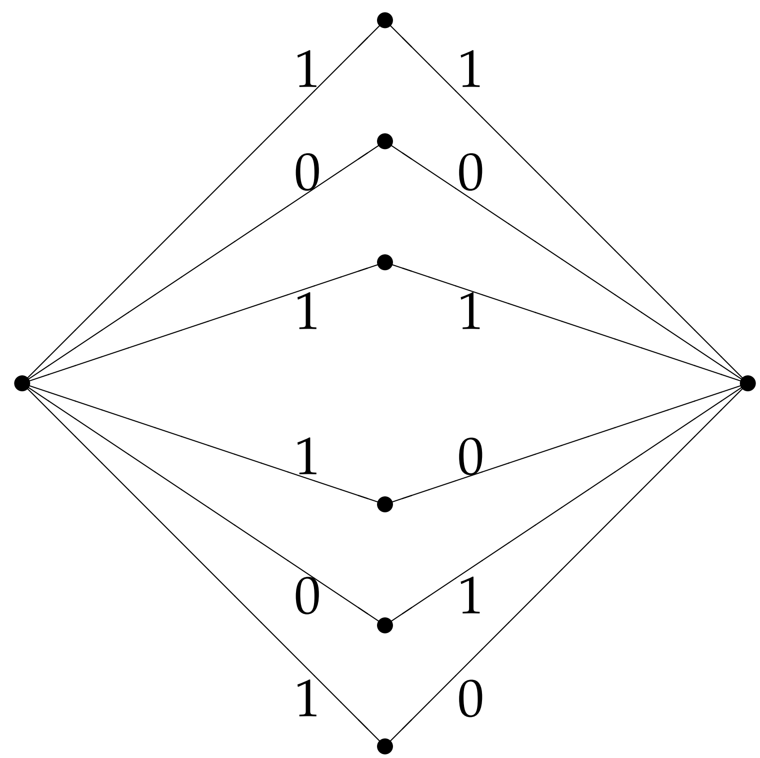
2-cover of srg(81,20,1,6)




srg(81,20,1,6)     unique Brouwer-Haemers

Edges of Brouwer-Haemers graph are labelled 0/1

Condition on labelling of edges in Brouwer-Haemers graph:




For 3 paths: edges of the path have the same label.

Weaker condition:
For 1, 3 or 5 paths: edges of the path have the same label.

Equivalent condition:
Sum of labels of the 12 edges of a $K_{2,6}$ is $\equiv 1 \pmod 2$.

For each of 2430 pairs of non-adjacent vertices in Brouwer-Haemers graph we have
a linear equation over $GF(2)$ in 810 unknowns.

Gaussian elimination:
There are no solutions.

A distance regular graph with intersection array $\{20,18,3,1;\ 1,3,18,20\}$ does not exist.

An srg(162,21,0,3) does not have an automorphism $g$ with order 2 and Fix$(g) = \emptyset$.

An srg(162,21,0,3) can not be vertex transitive.

Gavrilyuk mention yesterday: every *Cayley* mixed Moore graph of order $\leq 485$ is known (proved by Erskine).
The feasible mixed Moore graph of order 486 can not be vertex transitive, as srg(162,21,0,3) is a quotient.

Similarly, the following intersection arrays can be excluded:

1. $\{22, 21, 3, 1;\ 1, 3, 21, 22\}$
   2-cover of srg(100,22,0,6)

2. $\{32, 27, 6, 1;\ 1, 6, 27, 32\}$
   2-cover of srg(105,32,4,12)

3. $\{56, 45, 12, 1;\ 1, 12, 45, 56\}$
   2-cover of srg(162,56,10,24)

Case 2 was excluded by Soicher 2017.
Case 3 was excluded by Brouwer.